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A THEORETICAL ANALYSIS IN CHOOSING BETWEEN PROFIT-LOSS SHARING AND INTEREST-BASED CONTRACTS: A SIMPLE GAME MODEL

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ABSTRACT

Purpose — This study aims to investigate how participants decide between profit-loss sharing contracts (PLSC) and interest-based contracts (IBC) in an interactive environment. PLSC and IBC are two interesting financial arrangements. They are similar in that both transfer money from people who have excess money to those who are in need, but they are extremely different in sharing risks between participants.

Design/Methodology/Approach — The participants' profits change based on their role (as an investor or entrepreneur) and the selected contract because the contracts are entirely different in sharing or shifting risks. Thus, in the first step, various profit functions are constructed that differ according to each party's role and major factors relevant to each contract. Afterwards, a mathematical game model (GM) is developed to consider the parties' interaction concurrently. A numerical example also verifies the results.

Findings — The results show that the business output level, auditing costs, collateral related costs, and market conditions or state of the economy (SoE) are major factors in deciding between IBC and PLSC.

Originality/Value — This research sets up various profit functions (based on players' roles and contracts), enriching them by the contract-related factors and SoE and developing a GM. The existing literature focuses on the investor's optimal contract, while concluding a contract needs the mutual consent of the involved parties, not only the investor's inclination.

Practical Implication — This research provides a guideline for 'parties' share in PLSC' and generally accepted auditing standards for auditing PLSC.

Keywords — Auditing, Financial contract, Game model, Interest-based contract, Profit-loss sharing

Article Classification — Research paper

INTRODUCTION

Profit-loss sharing contracts (PLSC) and interest-based contracts (IBC) are two financial arrangements with interesting features. They are unique in transferring money from investors to entrepreneurs but are extremely different in sharing risks of the underlying business between the participants involved. By applying PLSC, parties agree to pool their resources (money, machinery, labour, management's ability, and so forth) in order to get a pre-determined share of the business profit (or loss if any) (Siddiqi, 1985; Al-Suwailem, 2002; Iqbal & Llewellyn, 2002). As the business is exposed to a variety of risks such as insolvency, competitive and reputational risks, technology changes, and political and market demand risks, the entrepreneur can mitigate the risks by choosing PLSC in financing a business. Meanwhile, the investor may prefer to get a likely higher return by applying a sharing contract as opposed to a lower fixed return based on a fixed payment contract. Contrary to PLSC, by adopting an IBC, the entrepreneur must pay the loan plus interest for financing his business. Since the interest rate in IBC is determined exogenously and is independent of the actual business return, the entrepreneur must tolerate all of the business risks to use IBC. The investor's outcome, however, is completely free of business risks in IBC.

PLSC and IBC are extremely different in the underlying business risks they face. Furthermore, due to their nature, their effect on the global economy is completely different. On the one hand, IBC has been recognised as the key factor in contributing to the recurrent economic crises in recent centuries due to the following main reasons:

1. Several studies verify that the mainstream finance system is the root of the recurrent economic crisis (Minsky, 1992; Allen, 1993; Allais, 1999; Reinhart & Rogoff, 2010; Benes & Kumhof, 2012).
2. IBC has been a dominant contract in the global financial system for some 200 years (Temin, 2014).
3. The dichotomy between the financial market and the real sector is the main reason for economic instability in the IBC system (Thornton, 1802; Wicksell, 1936; Iqbal & Molyneux, 2005; Van Greuning & Iqbal, 2007; Shaikh, 2012; Askari *et al.*, 2014).

To sum up, the popularity of IBC in the financial system amplifies the dichotomy created by IBC, which subsequently leads to an assumption that IBC could be the root of repeated economic crises over recent centuries. PLSC with its tiny share in the global financial market, on the other hand, fundamentally removes the dichotomy by creating a direct link between the real sector and the money market (Khan, 1986; Iqbal & Molyneux, 2005; Beck *et al.*, 2013). It is believed that increased use of PLSC in the global financial market would be a remedy for costly recurrent economic crises (Al-Jarhi, 2017).

Considering the above discussion, the question is: how can we enhance PLSC and expand its contribution to the economy? By considering profit as the participants' main incentive in choosing between contracts, the next question is what are the key factors in changing the agents' profit through these contracts? It is acceptable that concluding any contract requires the mutual

acceptance of all involved parties. This means that signing either a PLSC or IBC is the result of the acceptance of the contract conditions, structure, terms and necessities by both the investor and entrepreneur. The participants' decision would, moreover, be affected by the difference in the participants' expected profit that originates from the specific nature of the contract, along with the participants' attitude to risks and their ability in risk management. Moreover, the state of the economy (SoE) at the time of production or yield, which reflects market demand conditions, significantly influences the financial returns for participants. It is expected, for instance, that a business produces a higher profit during a boom (B) period than other (NB) periods. Applying PLSC increases the investor's return more than IBC in a B period. This means that participants' decisions on contract use may change by considering the SoE.

Accordingly, the main objective of this paper is to analyse the parties' decision-making process in choosing between PLSC and IBC when they co-exist. More specifically, the study aims to answer the following questions,

1. What factors play a role in the parties' profits when they decide to use PLSC or IBC?
2. How does the impact of the key factors on the parties' decision-making process change due to a change in the SoE?
3. How can policymakers affect the promotion of PLSC and IBC?

This research answers the questions by applying an analytical method and developing a game model (GM). GM is a preferred mode to analyse this condition for two reasons, namely the parties fully interact in selecting a contract since concluding a contract requires the mutual acceptance of all parties, and GM is a preferred method to study the cooperation between intelligent, rational decision-makers (Myerson, 2013). In this regard, a two-player GM is developed to evaluate the participants' reactions in an interactive framework. In this model, a typical entrepreneur and a typical investor are faced with two strategies (PLSC and IBC). Participants must choose either IBC or PLSC. This means that their interactions produce eight profit functions. Each party's profit function is defined based on his role and the accepted contract. In non-cooperating cases, their profits are assumed to be zero.

Developing a GM is the first novelty of the study. The incumbent literature mostly ignores the existence of full interaction between parties in selecting a contract. Instead, they focus on the investor side and how they can overcome the asymmetric information of PLSC, whereas the investor's and entrepreneur's inclinations are both critical factors when discussing their decision-making process for entering into a contract (Dang, 2010; Elfakir & Tkiouat, 2015). Another novelty of this study is the setting up of eight profit functions tailored according to the type of contract, the participants' role and SoE. The existing literature focuses only on the optimal contract in the view of the investor to overcome the asymmetric information that inherently exists in PLSC (Elfakir *et al.*, 2020). However, associated profit functions are set up by using the logic applied by Wang *et al.* (2020). In this regard, a regular general form of profit function (i.e., a production function and a cost function) is developed, and it is enriched by including associated parameters for the key factor that affects the profit including auditing strategy, market demand, interest rate and PLSC ratio. Applying a general form instead of a

specific form enables the researchers to avoid limiting assumptions such as constant return to scale and increases the flexibility of the model. It is assumed that the business uses a unique production function, which is a function of the effort level and money capital. Although the type of contract can affect the form of the production function (through the investor's intervention on the business decision), this issue is ignored in order to hold the separation between ownership and control rights. According to Fama and Jensen (1983), this separation is necessary for the survival of any organisation.

Two points must be explained about the methodology adopted in this article. First, financial intermediaries are absent in this model because the financial market is nothing except a complex variety of contracts between lenders and borrowers (both representing financial institutions, individuals/investors/entrepreneurs and the government). Thus, it is possible to do a financing activity without financial institutions, but it is logically impossible to lend or borrow money without a specific contract. Additionally, this research analyses how agents freely decide between contracts in a bare global financial market. Accordingly, this model is based on randomly matching investors and entrepreneurs. In this case, eliminating the financial intermediaries from the model does not pose a particular problem for the results. Moreover, some studies discovered that the structure of banks is a severe preventive factor of PLSC improvement (Dar & Presley, 2000; Abalkhail & Presley, 2002). Eliminating the intermediary entities from the model equalises the conditions for the two contracts. However, the banking system or other financial intermediaries may be assumed in the role of a borrower or a lender in the model.

Secondly, it is believed that PLSC is an ideal Islamic financial arrangement, but it does not mean that its performance depends on users' beliefs (Dar & Presley, 2000; 'Uṣmānī, 2004; Mirakhor, 2012). Furthermore, evidence from the real economy verifies that the use of PLSC is not limited to the Islamic financial system. Therefore, this paper focuses on the structure of contracts and how they make profit for participants regardless of the participants' beliefs.

This paper is organised into six sections. After the introduction, the second section briefly reviews the existing literature. The methodology of the study is discussed in the third section and the mathematical GM is established in the fourth section. A numerical example is discussed in the next section to verify the parametrical results. The last section presents the concluding remarks and policy implications.

LITERATURE REVIEW

The first part of this review discusses the concepts of IBC and PLSC and their main differences, while the second part reviews the earlier studies that discussed the agents' decision-making process in choosing between two contracts.

Differences between IBC and PLSC

The main function of both IBC and PLSC is to transfer money from the agents who have excess money to the agents who are in need. In the case of investment, for instance, the investor injects money into a business that is operated by an entrepreneur. However, IBC and PLSC are extremely different in sharing risks between participants. IBC is a risk-shifting contract as it

allows the total business risk to be borne by the entrepreneur. PLSC, on the other hand, is a risk-sharing mode whereby both the investor and entrepreneur share in the risky return (Abdul-Rahman *et al.*, 2014; Othman *et al.*, 2017; Abdul-Rahman *et al.*, 2019; Abdul-Rahman & Gholami, 2020; Abdul-Rahman *et al.*, 2020).

Due to their natures, both PLSC and IBC suffer from some weaknesses. By adopting PLSC, the investor faces at least two key problems: (1) the entrepreneur's dishonesty in doing his best, known as the moral hazard problem (Diamond, 1984; Ashour, 1999; Dang, 2010); and (2) the entrepreneur's dishonesty in reporting the true realised business profit (Sadr & Iqbal, 2002; Sadr & Gholami, 2020). On the other hand, the investor's main concerns in adopting IBC are the entrepreneur's default risk, the entrepreneur's foreclosure option, and his opportunistic behaviour (Hasan, 1985; Trester, 1998). By adopting IBC, the entrepreneur bears all the business risks when the level of economic stress is high. Therefore, PLSC and IBC both have advantages and disadvantages for both the investor and the entrepreneur. Since market demand and economic conditions are stochastic variables for both agents, the positive and negative effects of these contracts are intensified by including these variables in the discussion.

However, both IBC and PLSC are successful contracts in practice. IBC is a popular and dominant contract in the current global financial market. The most popular example of an IBC is conventional government bonds. PLSC is also a known contract worldwide. Contrary to Fathoni and Suryani (2020), who showed that the application of PLSC in Sharīʿah-compliant banking system is not optimal, the share of PLSC in the banking systems of Iran and Sudan (two full-fledged Islamic banking systems) and in Indonesia, as an example of the dual banking system, is about 30 per cent. In Malaysia and Pakistan, where the financial markets operate within the dual banking system, the share of PLSC is rising rapidly (Ascarya & Rokhimah, 2008; Muda & Ismail, 2010; Farooq *et al.*, 2013). Moreover, various forms of PLSC such as crop sharing (Crane & Leatham, 1993; Dar, 1997), profit sharing in factories (Ellis & Smith, 2010), participation contracts in the oil industry (Ghandi & Lin, 2014), joint venture capital arrangements (Presley, 2000), and the financing of high-tech industries (Trester, 1998) have always been a practical paradigm of investment over the world.

Regarding their weaknesses, three costly strategies have been introduced to control the challenges faced by PLSC and IBC. The investor who applies IBC may control the entrepreneur's default risk by requiring valuable collateral. In the case of PLSC, the investor can apply a monitoring policy to reduce the moral hazard problem, and an auditing strategy to control the entrepreneur's dishonesty in reporting true profits. The effect of the cost to the investor of all the mentioned policies must be included in the model. Finally, due diligence policy is a tactic to overcome the adverse selection problem in both contracts and choosing a proper contract may mitigate the risk of SoE.

Previous Studies

Nienhaus (1983) and Hasan (1985) discussed the evolution of PLSC when it co-exists with IBC using a simple mathematical model in which more profitable contracts (PLSC or IBC) can survive or evolve over time. In this model, profit is posited as the driving force behind its

evolution. They reached different results under certain and uncertain conditions. However, their simple model does not allow examining of the participants' interactions. Accordingly, they failed to discuss the participants' behaviour in an interactive model.

Nabi (2013) and Elfakir and Tkiouat (2015) discussed the feasibility of *mushārakah* against the debt contract by introducing an incentive mechanism to overcome the moral hazard problem. They introduced different strategies to overcome PLSC's asymmetry problems. It means that they only focused on the investor's side, whereas both the investor's and the entrepreneur's reactions are important in concluding any contract. Elfakir *et al.* (2020), on the other hand, focused on reducing the moral hazard problem based on the diminishing PLSC concept. In the context of a game theoretical approach, by combining this concept with real options, they found that the real options cooperation could be sustained by forcing the entrepreneur to be honest in reporting profits. Elfakir *et al.* (2020) focused on two types of risk-sharing contracts, each differing in their structural composition. In contrast to their study, this research examines two contracts: one involving risk sharing and the other risk shifting. While risk sharing distributes potential losses among stakeholders, with each bearing a portion of the total risk, risk shifting transfers the entire risk from one party to another, eliminating exposure for the original party.

Therefore, reviewing the existing literature revealed that no study specifically addresses the agents' decision process between PLSC and IBC when they co-exist within the same financial system. This study tries to fill the gap by designing a set of profit functions (to avoid useless complexity) that change by contract and according to the agent's role.

METHODOLOGY

By assuming profit as the agents' main incentive in choosing a contract and that neither player can provide a sufficient amount of the output if each contributes alone, as the first step, several profit functions are specified, which change according to the contract and the players' roles. To specify the appropriate profit functions, it is assumed that a typical investor tends to invest his money in a small- or medium-sized business that is operated by a typical skilled entrepreneur. It should be noted that the profit maximiser agents meet each other at time t_0, but the profit is obtained at time t_1. The common instance is a crop-sharing contract in which the harvest time is different from the time of signing the contract. For simplicity, it is assumed that there are only two legally binding agreements, PLSC and IBC, The parties are completely familiar with the structure of the contracts and they know their profit is sensitive to the contract. Moreover, the parties know the strategies to mitigate the associated risks, and they are aware of the impact of SoE at time t_1 on their own return. For instance, the investor knows that his return would be less risky by applying IBC compared to PLSC. Another assumption is that the business output is a function of the capital (money capital) and the effort level (labour).

In order to capture the impact of SoE, two types of profit functions are specified. The first type reflects the profit of the business when the output price is high; and the other type when the output price is low. To include the output price in the model, the logic of Sugema *et al.* (2010) is used. This states that productivity shock (in this model, the price shock) as an uncertainty factor

affects the production function. Since market demand or future economic condition is an unpredictable variable, both parties use weighted average return of two periods. The weight here is the likelihood of a high or low price occurring.

Profit functions are set up by using the logic applied by Wang *et al.* (2020). In this regard, a regular general form of parametrical profit function (containing a production function and a cost function) is developed. It is enriched by including associated factors such as auditing strategy, market demand, interest rate and PLSC ratio that affect the participants' related profit. It is assumed that the business uses a unique production function, which is a function of the effort level and money capital. By applying a general form instead of a specific form of the production function, this research is able to avoid limiting assumptions such as constant return to scale. Moreover, this simplistic assumption increases the flexibility of the model. The next point is that this research ignores the role of the investor's effort on the production function in order to maintain a separation between ownership and control rights. It is because the investor's intervention in business decisions threatens the survivability of the business (Fama & Jensen, 1983).

In the next step, a GM is developed by specifying the profit functions to discuss the participants' profit sensitivity to key factors. In developing a mathematical GM, it is assumed that a typical investor will decide to invest (his money or capital) in a business that is being operated by a typical entrepreneur at time t_0. Since the products of the underlying business will be obtained at time t_1, the profit of each party is a random variable at time t_0. They then try to maximise their profit by choosing IBC or PLSC. This is, in fact, a two-player game wherein each can choose a strategy. Each player is then faced with four expected payoffs. Since players act simultaneously in choosing contracts, this is a static cooperative game, a type of one-shot game developed by Rabin (1993) in which parties agree to cooperate by a specific contract. However, as the entrepreneur has access to more information (such as true realised profit or the level of his own effort) than the investor, it is an incomplete information game.

To further clarify the participants' decision sensitivity to different factors, a numerical example is carried out by incorporating changes in those factors' value.

MODEL
This section consists of two parts: the first part relates to the introduction of the related profit functions based on the role of the parties in each contract and enriching them by incorporating related variables. The second part is dedicated to the development of a mathematical GM.

Profit Functions
General Form of a Profit Function when Both Parties Use IBC
Equations (1) and (2) are the general form of a typical entrepreneur's profit function when he applies IBC in B and NB periods, respectively. π_b^{IBCB} in Equation (1) refers to a typical borrower's profit when he operates a typical firm financed by IBC in a B period and π_b^{IBCNB} is his profit when he operates the firm in an NB period. Similarly, π_l^{IBCB} and π_l^{IBCNB} in Equations (3) and (4) refer to the typical investor's profit when he applies IBC to finance the firm in B and

NB periods, respectively. The rest of the variables and parameters' definitions are provided below.

$$
\begin{vmatrix}
\text{B} & \pi_b^{IBCB} = P^B . q^{IBC}(e^{IBC}, F) - c_b^{IBC}(e_b) - \bar{r}f - f - \bar{\psi}Q & (1) \\
\text{NB} & \pi_b^{IBCNB} = P^{NB} . q^{IBC}(e^{IBC}, F) - c_b^{IBC}(e_b) - \bar{\psi}Q - \varpi Q & (2)
\end{vmatrix}
$$

$$
\begin{vmatrix}
\text{B} & \pi_l^{IBCB} = \bar{r}f & (3) \\
\text{NB} & \{\pi_l^{IBCNB} = Q - f , \qquad if \ \emptyset = 1 & (4)
\end{vmatrix}
$$

where
π: profit
Subscripts b and l are for the borrower and lender, respectively
Superscript IBC for contract, B for B period, NB for NB period
$c_b^{IBC}(e_b)$: the borrower's effort cost in operating the business
q^{IBC}: output level by applying IBC
$\bar{r}$: pre-determined interest rate
F: total required fund $(f_1 + f)$ where f: the lender's contribution in funding the business, and f_1: the borrower's contribution in the required capital
Q: the value of the collateral, $\bar{\psi}$: the cost of providing collateral, ϖ: probability of losing collateral
P^B and P^{NB}: the business output price in B and NB periods
$\bar{w}_A$: auditing cost rate

According to Equation (1), the entrepreneur's profit when he applies IBC in a B period is the business total revenue $(P^B . q^{IBC}(e^{IBC}, F))$ minus the business total costs (disutility of the borrower's effort, $c_b^{IBC}(e_b)$, plus the principal of the borrowed money and its costs $\bar{r}f + f$, and the cost of providing collateral $(\bar{\psi}Q)$. Output, $q^{IBC}(e^{IBC}, F)$, is a function of the entrepreneur's effort and money capital. The borrower's effort function is assumed to increase concavely. This means that a higher level of the borrower's effort will result in more products but this positive effect rapidly declines with additional effort. Then, disutility of the borrower's effort (his cost) function in general form satisfies the normal condition (concavity to the level of effort) as wage in a labour economy (Gill & Prowse, 2019). The entrepreneur's profit in an NB period is almost the same as in the B period, except for the term ϖQ, i.e., probalility of losing collateral. Equations (3) and (4) say that the lender takes a pre-determined return $(\bar{r}f)$ in B and $(Q - f)$ in an NB period. This means that the lender's return in an NB period would be equal to the excess value of the collateral from the loan.

As SoE is a stochastic variable, the Expected Profit (EP) of each party is simply a weighted average of their profits in the two periods. The weight here is the probability of a B condition occurring. In Equation (6), $E(\pi_b^{IBC})$ stands for the borrower's EP and ρ and $(1 - \rho)$ for the probability of B or NB occurring, respectively.

$$E(\pi_b^{IBC}) = \rho(\pi_b^{IBCB}) + (1-\rho)(\pi_b^{IBCNB}) \tag{5}$$

$$E(\pi_b^{IBC}) = q^{IBC}(e^{IBC}, F)[\rho P^B + (1-\rho)P^{NB}] - [c_b^{IBC}(e_b) + (\bar{\psi} + \varpi - \rho\varpi)Q + (1+\bar{r})\rho f] \tag{6}$$

Equation (6) is obtained by plugging in for π_b^{IBCB} and π_b^{IBCNB} from Equations (1) and (2) and using simple mathematics (**Appendix A.1**). It shows that the entrepreneur's EP by using IBC is the weighted average of the business expected revenue in two periods (the first bracket) minus total cost (second bracket). Total cost includes the cost of providing collateral plus the likelihood of losing the collateral as well as repayment of the loan and its interest. The point here is that SoE has impact on both the entrepreneur's revenue and cost.

The lender's expected return when he uses IBC in two SoEs is as follows (**Appendix A.2**):

$$E(\pi_l^{IBC}) = \rho(\pi_l^{IBCB}) + (1-\rho)\pi_l^{IBCNB}) \tag{7}$$
$$= (\rho\bar{r} + \rho - 1)f + (1-\rho)Q \tag{8}$$

According to Equation (8), the investor's EP when he uses IBC is clearly sensitive to the worthiness of the collateral, the amount of the loan (his contribution), the interest rate, and the likelihood of economic prosperity but not to the output level or output price. It can be easily shown that the investor's EP is always positive if,

$$(\pi_l^{IBC}) > 0 \quad if \quad (\rho\bar{r} + \rho - 1)f + (1-\rho)Q > 0$$

Finally,
$$Q > \frac{(1 - \rho\bar{r} - \rho)}{(1-\rho)}f \tag{9}$$

Equation (9) shows that the minimum value of the collateral depends on f, $\bar{r}$ and ρ. A rise in ρ for any level of the interest rate decreases the fraction behind f on the right hand side (RHS) of Equation (9). It shows that this fraction itself is non-negative for any $0 < \rho < 1$ and decreases by an increase in ρ (**Appendix A.3**). It means that the amount of the required collateral decreases as the likelihood of the B period increases. In total, Equation (9) shows that the collateral plays a key role in the decision-making process of the investor who applies IBC.

General Form of the Payoff Function when the Parties Apply PLSC
Equations (10) , (11), (15) and (16) show the general form of a typical entrepreneur's and investor's profit functions when they use PLSC in B and NB periods. π_b^{PLSCB} and π_b^{PLSCNB} in Equations (10) and (11) refer to the entrepreneur's profit in B and NB periods, respectively. Additionally, $\alpha\epsilon(0,1)$ is the borrower's pre-determined share of the total business revenue, f_1 is the investor's contribution in the business, and $(F - f_1)$ is the borrower's self-financing.

$$\text{B} \qquad \pi_b^{PLSCB} = \alpha[P^B q^{PLSC}(e(e_b), F)] - c_b^{PLSC}(e_b) - (F - f_1) \qquad (10)$$

$$\text{NB} \qquad \pi_b^{PLSCNB} = \alpha[P^{NB} q^{PLSC}(e(e_b), F)] - c_b^{PLSC}(e_b) - (F - f_1) \qquad (11)$$

Comparing Equations (10) and (11) with (1) and (2) shows how the contract affects the profit functions. These differences stem from the fact that parties share in the revenue not in the profit. Subtracting each party's relevant costs from their own revenue (as a share of the business revenue) produces a profit that is comparable with the profit of the entrepreneur who uses IBC, i.e., $\bar{r}f$. As is clear from Equations (10) and (11), the entrepreneur's profit is his own revenue (the pre-determined share of the business revenue) minus his costs and his self-financing.

The average profit for the entrepreneur who uses PLSC in B and NB periods is obtained by plugging in Equation (12) from Equations (10) and (11) (**Appendix B**).

$$E(\pi_b^{PLSC}) = \rho \pi_b^{PLSCB} + (1 - \rho) \pi_b^{PLSCNB} \qquad (12)$$

$$E(\pi_b^{PLSC}) = \alpha[\rho P^B + (1 - \rho)P^{NB}]q^{PLSC}(e(e_b), F)] - c_b^{PLSC}(e_b) - (F - f_1) \qquad (13)$$

The first term on the RHS of Equation (13) is the entrepreneur's revenue, α share of total revenue. This term is always non-negative because the revenue at the minimum level is zero. The last two terms in the RHS of this Equation are the entrepreneur's effort cost and his self-money contribution, which are always positive. However, they are always negative by considering the minus in front. Hence, the entrepreneur's EP (not revenue) is positive if the entrepreneur's revenue (α share of total revenue) exceeds his costs plus his contribution.

Equalizing the entrepreneur's EP to zero gives the equilibrium path for his share of total revenue (α).

$$E(\pi_b^{PLSC}) = \alpha[\rho P^B + (1 - \rho)P^{NB}]q^{PLSC}(e(e_b), F)] - c_b^{PLSC}(e_b) - (F - f_1) = 0$$

$$\text{Thus,} \qquad \alpha^* = \frac{c_b^{PLSC}(e_b) + (F - f_1)}{[\rho P^B + (1 - \rho)P^{NB}]q^{PLSC}(e(e_b), F)]} \qquad (14)$$

According to Equation (14), the entrepreneur's EP becomes zero when α is equal to the sum of the entrepreneur's total costs and his contribution divided by the business' expected total revenue. This share (α^*) is the minimum share that is tolerated by the entrepreneur (breakpoint). This means that for any share less than α^*, the entrepreneur's profit becomes negative. As a numerical example, let's assume that the expected revenue of a business is 10 units while each of the entrepreneur's cost and his contribution is 2 units. In this case, $\alpha^* = 40$ per cent is the breakpoint. In other words, the entrepreneur's share must never be lower than 40 per cent of the business total revenue; otherwise, his EP would be negative. The equation clearly shows that an increase in the entrepreneur's contribution $(F - f_1)$ pushes the breakpoint higher. As an example, when the entrepreneur's contribution is zero, in the case of *muḍārabah,* the entrepreneur's breakpoint shifts down, as the second term in the numerator becomes zero. Then, almost the total business revenue must be paid to the investor in *muḍārabah*.

Equations (15) and (16) show the investor's profit when he applies PLSC in B and NB periods, respectively. π_l^{PLSCB} and π_l^{PLSCNB} in these equations refer to the investor's profit in B and NB periods, respectively.

B $\pi_l^{PLSCB} = (1-\alpha)[P^B q^{PLSC}(e(e_b), F)] - c_l^{PLSC}(e_l) - \overline{w}_A F - f_1$ (15)

NB $\pi_l^{PLSCNB} = (1-\alpha)[P^{NB} q^{PLSC}(e(e_b), F)] - c_l^{PLSC}(e_l) - \overline{w}_A F - f_1$ (16)

The term $c_l^{PLSC}(e_l)$ in the RHS of Equations (15) and (16) denotes the investor's effort/cost to monitor the entrepreneur to overcome the moral hazard problem. Moreover, it includes the auditing strategy that the investor implements to force the entrepreneur to report the true realised revenue, assumed as a percentage $(\overline{w}_A)$ of the total required money (F). For the sake of avoiding complexity, this research uses F instead of cash flows. Finally, the investor requires the entrepreneur to finance a part of F to reduce the moral hazard problem. By applying these three strategies, the investor may have a greater control on the information problem that inherently exists in PLSC.

Another important point about the investor's profit in Equations (15) and (16) is that his profit is not comparable with the lender's return in IBC. This is because the former contains the amount of the loan, while the latter does not. To make them comparable, the lender's contribution f_1, must be subtracted from the RHS.

Plugging in Equation (17) for π_l^{PLSCB} and π_l^{PLSCNB} in Equations (15) and (16) gives, the lender's Expected Profit:

$$E(\pi_l^{PLSC}) = \rho\pi_l^{PLSCB} + (1-\rho)\pi_l^{PLSCNB}$$ (17)

$$E(\pi_l^{PLSC}) = (1-\alpha)(\rho P^B + (1-\rho)P^{NB})q^{PLSC}(e(e_b), F) - c_l^{PLSC}(e_l) - \overline{w}_A F - f_1$$ (18)

The first term on the RHS of Equation (18) is the investor's share of the business' expected revenue, which is a positive variable (as mentioned before). The sum of the last three terms is always positive. However, considering the minus sign in front of them converts them to negative. Based on this equation, the investor's EP is greater than zero if his average profit exceeds his costs plus his principal contribution. Rearranging Equation (18) based on $(1-\alpha)$ gives,

$$(1-\alpha)^* = \frac{c_l^{PLSC}(e_l) + \overline{w}_A F + f_1}{(\rho P^B + (1-\rho)P^{NB})q^{PLSC}(e(e_b), F)}$$ (19)

In Equation (19), $(1-\alpha)^*$ is the minimum share tolerated by the investor (breakpoint). In other words, any share less than $(1-\alpha)^*$ negates the investor's expected profit. As a numerical example, let's consider the average business profit is about 10 units while the investor's total cost for auditing, monitoring in addition to the lender's contribution is about 3 units and his contribution is 5. Then, any share less than 80 per cent of the total revenue negates the investor's expected profit.

So far, four profit functions for two agents and two contracts have been presented. **Table 1** summarises the results.

Table 1: The Agents' Expected Profit

The Entrepreneur	$E(\pi_b^{IBC}) = q^{IBC}[\rho P^B + (1-\rho)P^{NB}] - [c_b^{IBC}(e_b)$ $+ (\bar{\psi} + \varpi - \rho\varpi)Q + (1+\bar{r})\rho f]$ $E(\pi_b^{PLSC}) = \alpha[[\rho P^B + (1-\rho)P^{NB}]]q^{PLSC} - c_b^{PLSC}(e_b) - (F - f_1)$
The Investor	$E(\pi_l^{IBC}) = (\rho\bar{r} + \rho - 1)f + (1-\rho)Q$ $E(\pi_l^{PLSC}) = (1-\alpha)[[\rho P^B + (1-\rho)P^{NB}]]q^{PLSC} - c_l^{PLSC}(e_l) - \bar{w}_A F - f_1$

Source: Authors' own

Table 1 clearly shows how the parties' EP changes according to the contract used and the participant's role.

Game Model

Table 2 shows a matrix of the parties' EP in the normal form of a GM.

Table 2: Matrix of Parties' EP

The Entrepreneur

The Investor		IBC		PLSC	
	IBC	$E(\pi_l^{IBC})$,	$E(\pi_b^{IBC})$	0 ,	0
	PLSC	0 ,	0	$E(\pi_l^{PLSC})$,	$E(\pi_b^{PLSC})$

Source: Authors' own

In the above matrix, the investor is a row and the entrepreneur is a column player. The matrix contains four cells and each cell has two elements. The first element in each cell is the payoff (profit) of the row player and the second element is the column player's payoff. If both players use IBC, the investor's EP is $E(\pi_l^{IBC})$, the first element of the first cell, and the entrepreneur's EP is $E(\pi_b^{IBC})$, the second element of the first cell. However, if both players apply PLSC, the investor's EP is $E(\pi_l^{PLSC})$, while the entrepreneur's EP is $E(\pi_b^{PLSC})$ (the first and second elements of the fourth cell). Finally, if the investor persists on IBC while the entrepreneur changes his strategy to PLSC, both get zero profit (the first and the second element of the second cell) and vice versa. The parties' payoff becomes zero due to the assumption of 'no deal, no payoff'. Future research may relax this assumption to get more realistic results.

The first point about the above matrix is that the sum of the elements in each cell is not always zero. Let's consider a situation in which both players adopt IBC; in that case, the sum of their EP is:

$$E(\pi_l^{IBC}) + E(\pi_b^{IBC}) = (\rho\bar{r} + \rho - 1)f + (1 - \rho)Q + q^{IBC}[\rho P^B + (1 - \rho)P^{NB}] - [c_b^{IBC}(e_b) +$$
$$(\bar{\psi} + \varpi - \rho\varpi)Q + (1 + \bar{r})\rho f] \neq 0. \tag{20}$$

The result of Equation (20) is rarely equal to zero because it contains so many different and independent variables. This means that this game is often a win-win cooperative game and the agents in this game may improve their payoffs by cooperating and engaging in a deal. The second point that arises from the structure of the matrix is that the main difference between PLSC and IBC is not just the distribution of profit (or loss) but in distributing income between involved participants.

The Equilibrium

Three equilibriums are possible for any game; two Nash Equilibrium (NE) and an equilibrium for a mixed strategy. NE is a situation in which none of the players has an incentive to change their strategy while the opponent persists in his choice (Dutta, 1999). Stated differently, in an NE no one can get more payoff by changing his strategy. In addition to pure strategies, any game has a mixed strategy too. An equilibrium is a strictly NE when both players' EP becomes greater than zero. A mixed strategy means how likely a player is to play each of the possible strategies (Osborne, 2004). Alternatively, if a player repeats a game for a hundred times, for instance, what is the frequency of each of the chosen strategies? A mixed strategy applies often in a situation where players decide repeatedly on the available strategies.

The following parts discuss the necessary conditions under which PLSC and IBC are strictly NE.

IBC as a Pure NE

As said before, a necessary condition for IBC to be a strictly NE is that 'both players' EP becomes greater than zero'. In **Table 2**, for instance, when the investor changes his strategy from IBC to PLSC while the entrepreneur persists on the IBC, both players' EP becomes zero. In this case, if the investor's EP is bigger than zero, i.e., $(\rho\bar{r} + \rho - 1)f + (1 - \rho)Q > 0$, changing the strategy will make him suffer. In mathematical terms, IBC is strictly NE if both players' EP simultaneously becomes greater than zero, as shown in the system of inequality (21):

$$E(\pi_l^{IBC}) = (\rho\bar{r} + \rho - 1)f + (1 - \rho)Q > 0$$
$$E(\pi_b^{IBC}) = q^{IBC}[\rho P^B + (1 - \rho)P^{NB}] - [c_b^{IBC}(e_b) + (\bar{\psi} + \varpi - \rho\varpi)Q + (1 + \bar{r})\rho f] > 0 \tag{21}$$

Changing the system of inequality (21) to the system of equations and rearranging them based on $\bar{r}$ (as a key variable in IBC) and using simple mathematics gives (**Appendix C**):

$$\bar{r}_l = \frac{\rho(\rho - 1)(Q - f)}{f} \qquad\qquad \text{The Investor} \tag{21a}$$

$$\bar{r}_b = \frac{\{q^{IBC}[\rho P^B + (1 - \rho)P^{NB}] - c_b^{IBC}(e_b) - (\bar{\psi} + \varpi - \rho\varpi)Q\}}{\rho f} - 1 \qquad\qquad \text{The Entrepreneur} \tag{21b}$$

The fraction on the RHS of Equation (21a) determines the minimum interest rate tolerable by the investor. It means that any interest rate bigger than the fraction $(\frac{\rho(\rho-1)(Q-f)}{f})$ makes the investor's EP greater than zero. It means that for each $\bar{r} > \bar{r}_l$, IBC is a strictly preferable contract for the investor. Moreover, the fraction clearly shows that the investor's minimum return ($\bar{r}_l$) is independent of the business' actual return, while it would be directly affected by ρ, Q and f. As the fraction shows, the term $\rho(\rho-1)$ in the numerator is always negative because $0 < \rho < 1$, $\rho^2 < \rho$. The denominator, which is the investor's contribution (loan), is always positive too. Then, the sign of the fraction is the sign of $(Q - f)$. If $Q = f$ the fraction becomes 'equal to zero'. Then, for any amount of Q greater than f, the investor's minimum acceptable interest rate will be negative. This means that the economic condition does not matter for the investor who has valuable collateral from the entrepreneur.

In the case of the entrepreneur, Equation (21b) states that for any condition in which the exogenous interest rate, $\bar{r}$, is less than the fraction on the RHS, the entrepreneur's EP becomes greater than zero. The denominator ρf is always non-negative. The numerator contains the expected business revenue minus the entrepreneur's specific costs (including effort cost and the collateral-related costs). The fraction indicates that the entrepreneur's EP depends on the factors affecting the investor's EP, the total business revenue, the borrower's effort costs, and the collateral-related costs. The sign of the fraction could be negative, positive or zero based on the economic condition and the cost's structure.

Since Equations (21a) and (21b) are the minimum and the maximum level of the interest rate that is tolerable by the two parties, it is possible to define a space between these two rates called IBC Improvement Space ($IITS_r$ or simply IITS). To analyse the sensitivity of the IITS to ρ, f, Q, ϖ and $\bar{\psi}$ let's assume:

$$IITS_r = \bar{r}_b - \bar{r}_l = \frac{\{q^{IBC}[\rho P^B + (1-\rho)P^{NB}] - c_b^{IBC}(e_b) - (\bar{\psi} + \varpi - \rho\varpi)Q\}}{\rho f} - \frac{\rho(\rho-1)(Q-f)}{f} - 1 \qquad (21c)$$

Taking a partial derivative of the space ($IITS_r$) with respect to the mentioned variables, gives the sensitivity of the space to the key factors. The results from Matlab Software are shown in **Table 3** (**Appendix D** indicates the software results).

Table 3 shows how a change in value of the variables changes $IITS$. The first row indicates the sensitivity of IITS to ρ. The first order condition (the derivation of IITS to ρ and making the results equal to zero) gives two extremums. This means that for the two values of ρ the space is equal to zero. Since the second derivation (second order condition) is negative, it verifies that the function has a maximum and IITS is an inverse U-shape with respect to ρ. The interpretation is that with the increase in the likelihood of economic prosperity (for points after maximum), IITS will decline. Differently speaking, IBC is a harder contract (or IITS becomes narrower) when the likelihood of a boom period happening is remarkably high.

Table 3: The Partial Derivation Results

Row	Derivation	Sign
1	$\frac{\partial IITS_r}{\partial \rho} =$ $\frac{\varpi Q + q^{IBC}(P^B - P^{BN})}{\rho f} + \frac{(1-\rho)(Q-f)}{f} - \frac{\rho(Q-f)}{f} +$ $\frac{c_b^{IBC}(e_b) + Q(\varpi + \bar{\psi} - \rho\varpi) - q^{IBC}(\rho P^B + (1-\rho)P^{BN})}{f\rho^2}$	$\frac{\partial IITS_r}{\partial \rho} = 0$ contains two roots, $\frac{\partial(\frac{\partial IITS}{\partial \rho})}{\partial \rho} < 0$
2	$\frac{\partial IITS_r}{\partial f} = \frac{\rho(\rho-1)}{f} + \frac{c_b^{IBC}(e_b) + Q(\varpi + \bar{\psi} - \rho\varpi) - q^{IBC}(\rho P^B + (1-\rho)P^{BN})}{\rho f^2} + \frac{\rho(\rho-1)(Q-f)}{f^2}$	$\frac{\partial IITS_r}{\partial f} = 0$ no root $\frac{\partial IITS_r}{\partial f} < 0$ $\frac{\partial(\frac{\partial IITS}{\partial f})}{\partial f} > 0$
3	$\frac{\partial IITS_r}{\partial Q} = \frac{\rho(1-\rho)}{f} - \frac{\varpi(1-\rho) + \bar{\psi}}{\rho f}$	$\frac{\partial IITS_r}{\partial Q} = 0$ no root, $\frac{\partial IITS_r}{\partial Q} < 0$ $\frac{\partial(\frac{\partial IITS}{\partial Q})}{\partial Q} = 0$
4	$\frac{\partial IITS_r}{\partial \varpi} = \frac{(\rho-1)Q}{\rho f}$	$\frac{\partial IITS_r}{\partial q} < 0$ always negative $\frac{\partial(\frac{\partial IITS}{\partial om})}{\partial om} = 0$
5	$\frac{\partial IITS_r}{\partial \bar{\psi}} = -\frac{Q}{\rho f}$	$\frac{\partial IITS_r}{\partial \bar{\psi}} < 0$ always negative $\frac{\partial(\frac{\partial IITS}{\partial \bar{\psi}})}{\partial \bar{\psi}} = 0$

Source: Authors' own

The second row of the table depicts how IITS changes by a change in the loan. The first order condition shows that there is no extremum point for the IITS with respect to f. However, the sign of the first derivative is almost negative because three fractions on the RHS are always negative. The first and the third fraction are always negative because $\rho^2 < \rho$ for $0 < \rho < 1$. The second fraction is logically negative because in a normal business, the expected business revenue often exceeds the borrower's cost (effort and collateral costs). Then $\frac{\partial IITS_r}{\partial f} < 0$ shows that IITS declines for any rise in f. In other words, an increase in the amount of the loan narrows the gap and makes IBC a harder contract. In the same way, it shows that an increase in Q for the low level of ρ widens IITS (third row of the table). It is because the sign of the first fraction on the RHS is negative due to $\rho^2 < \rho$ for $0 < \rho < 1$ and the second fraction's sign is negative due to the negative sign on its front. The justification is that providing the collateral is costly for the entrepreneur. For the same reason, the results are the same for the cost of providing collateral ($\bar{\psi}$) and the likelihood of losing collateral. Therefore, an increase in the value of the collateral and its related costs narrows IITS and makes IBC a harder contract.

PLSC as a Pure NE
Like the IBC case, PLSC is strictly NE when both parties get a positive payoff simultaneously by applying PLSC. Consider **Table 3** again whereby the lender changes his strategy from PLSC to IBC while the borrower persists on the PLSC. In this case, both players' EP becomes zero. In mathematical terms, PLSC is strictly NE if:

$$E(\pi_l^{PLSC}) = \alpha_l[\rho P^B + (1-\rho)P^{NB}]q^{PLSC} - c_l^{PLSC}(e_l) - \overline{w}_A F - f_1 > 0 \qquad (22)$$

$$E(\pi_b^{PLSC}) = \alpha_b[\rho P^B + (1-\rho)P^{NB}]q^{PLSC} - c_b^{PLSC}(e_b) - (F - f_1) > 0 \qquad (23)$$

Equalising the above equations to zero and rearranging them based on the parties' share of the revenue (as a key decision variable in PLSC), gives Equations (22a) and (23a).

$$\alpha_l = \frac{[c_l^{PLSC}(e_l) + \overline{w}_A F + f_1]}{[\rho P^B + (1-\rho)P^{NB}]q^{PLSC}} \qquad (22a) \qquad \alpha_b = \frac{[c_b^{PLSC}(e_b) + (F - f_1)]}{[\rho P^B + (1-\rho)P^{NB}]q^{PLSC}} \qquad (23a)$$

Parameters α_l and α_b are the investor's and the entrepreneur's affordable (minimum) share, respectively. This means that the lender does not accept any share lower than α_l and in the same way, the borrower does not tolerate any share lower than α_b. Therefore, any share more than these two extremes ensures that both parties' payoffs are bigger than zero and PLSC is a possible contract.

There are a few points about Equations (22a) and (23a). First, if the investor's contribution to the business is more than the entrepreneur's contribution ($\frac{f_1}{F} > 0.5$), the investor's minimum share must be greater than the borrower's share, $\alpha_l \geq \alpha_b$. The reason is that a minimum share for the investor must compensate his related costs in addition to repaying his capital contribution (f_1). Second, since α_l and α_b are the minimum share (breakpoint), their sum will be less than one, i.e., $\alpha_l + \alpha_b < 1$. In this case, the space that is created by $[1 - (\alpha_l + \alpha_b)]$ is called Profit-Loss Sharing Improvement Space (PITS). Both parties strictly prefer PLSC inside this space. Widening the space means a higher level of the PLSC possibility and vice versa. Inserting α_l and α_b from Equation (22a) and (23a) in PITS shows that the value of PITS depends on several factors (as per Equation 24).

$$\begin{aligned} PITS &= [1 - (\alpha_l + \alpha_b)] \\ &= 1 - \left(\frac{[c_l^{PLSC}(e_l) + \overline{w}_A F + f_1]}{[\rho P^B + (1-\rho)P^{NB}]q^{PLSC}} + \frac{[c_b^{PLSC}(e_b) + (F - f_1)]}{[\rho P^B + (1-\rho)P^{NB}]q^{PLSC}}\right) \\ &= 1 - \frac{c_l^{PLSC}(e_l) + c_b^{PLSC}(e_b) + (\overline{w}_A + 1)F}{[\rho P^B + (1-\rho)P^{NB}]q^{PLSC}} \qquad (24) \end{aligned}$$

As Equation (24) shows, the size of PITS declines by an increase in the parties' effort costs, auditing costs and the amount of required funds. Interestingly, a rise in any of these costs (no matter from the investor's or entrepreneur's side) for any level of the business revenue and total costs, narrows the space. However, a rise in total business revenue widens the space.

Taking partial derivative of the space to each variable (ρ, $\overline{w}_A$, C_l, q and F) shows the space sensitivity to those factors. **Table 4** shows the results obtained by Matlab software (Software output is presented in **Appendix E**).

Table 4: The Derivation Results

Row	Derivation	Sign
1	$\dfrac{\partial PITS}{\partial \rho} = \dfrac{(P^B - P^{NB})(c_l^{PLSC}(e_l) + c_b^{PLSC}(e_b) + (\overline{w}_A + 1)F)}{q^{PLSC}(\rho P^B + (1-\rho)P^{NB})^2}$	$\dfrac{\partial PITS}{\partial \rho} > 0, \dfrac{\partial(\frac{\partial PITS}{\partial \rho})}{\partial \rho} < 0$
2	$\dfrac{\partial PITS}{\partial q^{PLSC}} = \dfrac{\left(c_l^{PLSC}(e_l) + c_b^{PLSC}(e_b) + (\overline{w}_A + 1)F\right)}{[\rho P^B + (1-\rho)P^{NB}]q^{PLSC^2}}$	$\dfrac{\partial PITS}{\partial q^{PLSC}} > 0, \dfrac{\partial(\frac{\partial PITS}{\partial q})}{\partial q} < 0$
3	$\dfrac{\partial PITS}{\partial F} = \dfrac{-(\overline{w}_A+1)}{[\rho P^B+(1-\rho)P^{NB}]q^{PLSC}}$	$\dfrac{\partial PITS}{\partial F} < 0, \dfrac{\partial(\frac{\partial PITS}{\partial F})}{\partial F} = 0$
4	$\dfrac{\partial PITS}{\partial \overline{w}_A} = -\dfrac{F}{[\rho P^B + (1-\rho)P^{NB}]q^{PLSC}}$	$\dfrac{\partial PITS}{\partial \overline{w}_A} < 0, \dfrac{\partial(\frac{\partial PITS}{\partial \overline{w}_A})}{\partial \overline{w}_A} = 0$
5	$\dfrac{\partial PITS}{\partial c_l^{PLSC}(e_l)} = -\dfrac{1}{[\rho P^B+(1-\rho)P^{NB}]q^{PLSC}}$	$\dfrac{\partial PITS}{\partial c_l^{PLSC}(e_l)} < 0, \dfrac{\partial(\frac{\partial PITS}{\partial c_l^{PLSC}(e_l)})}{\partial c_l^{PLSC}(e_l)} = 0$

Source: Authors' own

In the first row of **Table 4**, the variables in the numerator and denominator are all positive. Then the sign of $\frac{\partial PITS}{\partial \rho}$ is positive whenever the price in the B period is greater than the NB period. The sign of $\frac{\partial PITS}{\partial q}$ is also positive. This means that an increase in the likelihood of B or an increase in the business output level widens the space and makes PLSC a more feasible contract. In the same line of the above discussion, it is shown that a rise in F, $\overline{w}_A$ and $c_l^{PLSC}(e_l)$ narrows PITS. In other words, an increase in the auditing cost and a rise in the lender's effort costs (to monitor the borrower) makes PITS a harder contract. It should be noted that the numerator in this fraction $\frac{-(\overline{w}_A+1)}{[\rho P^B+(1-\rho)P^{NB}]q^{PLSC}}$ is tiny compared to the denominator, in which case the effect of F is negligible. In sum, C_l, and F negatively affect PITS, while ρ, $\overline{w}_A$ and q have a positive impact on it.

NUMERICAL EXAMPLE

In the previous section, the sensitivity of PITS and IITS to various factors in parametric forms was discussed. To make it easier to understand how the mentioned spaces react to a change in different factors, a numerical example has been conducted. Consider again Equations (21.c) and (24) and let's assume the following numbers in **Table 5**.

Although we set fixed digits for a few variables such as required fund, total output, the borrower's and the lender's related costs, we allow other variables and figures to vary within a certain range. The likelihood of the B period and the interest rate, for instance, changes between zero and one and so on. We discuss the reaction of the IITS and PITS by a change in factors in question.

By substituting digits from **Table 5**, in Equations 21.c and 24, **Figures 1(a), 1(b), 1(c), 1(d), 1(e), 1(f)** show how PITS and IITS change with respect to the change in $\rho, \frac{Q}{f}, \overline{w}_A, \overline{\psi}, \varpi$ and q.

Table 5: Values for Variables

Variables Definition	Code	$ / %
Production function		
Total required fund (units)	F	200
The IBC lender's capital contribution (%)	f	80
The IBC borrower's contribution (endowment) (%)	k	20
The PLSC lender's capital contribution (%)	f_1	80
The PLSC borrower's contribution (endowment) (%)	k_1	20
Optimum output (PLSC)	q^{PLSC}	2000
Optimum output (IBC)	q^{IBC}	2000
Prices		
Interest rate	$\bar{r}$	(0, 1)
Output price in B period	P^u	1.2
Output price in NB period	P^d	1
Uncertainty		
Crisis probability	ρ	(0, 1)
Costs		
Cost of providing collateral (per cent of collateral)	$\bar{\psi}$	0.02
Probability of losing collateral (%)	ϖ	0.06
Auditing costs (%)	$\overline{w}_A$	0.05

Source: Authors' own

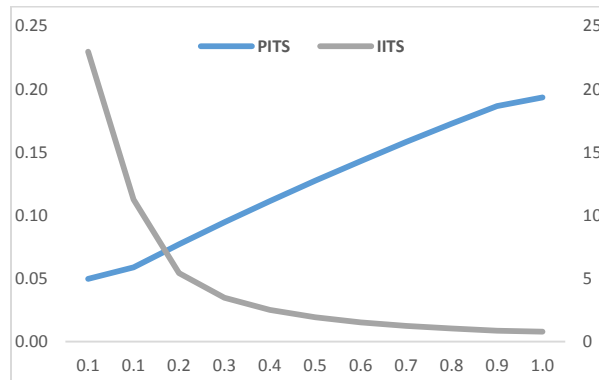**Figure 1(a) Sensitivity of IITS and PITS to ρ**

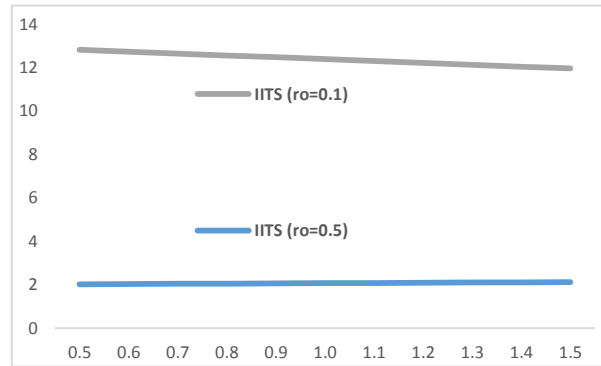**Figure 1(b) Sensitivity of IITS to $\frac{Q}{f}$ and ρ(ro)**

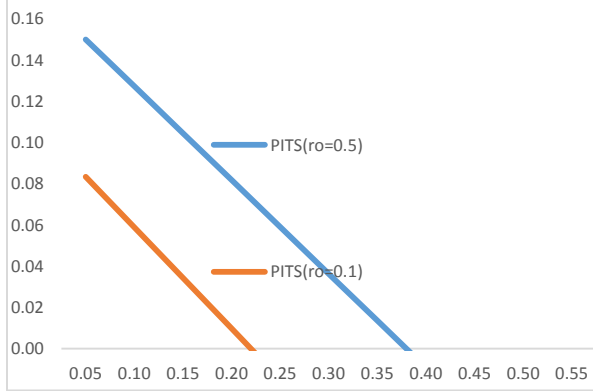**Figure 1(c) Sensitivity of PITS to $\bar{w}_A$ and ρ(ro)**

Figure 1(d) Sensitivity of IITS to $\bar{\bar{\psi}}$ and ρ(ro)
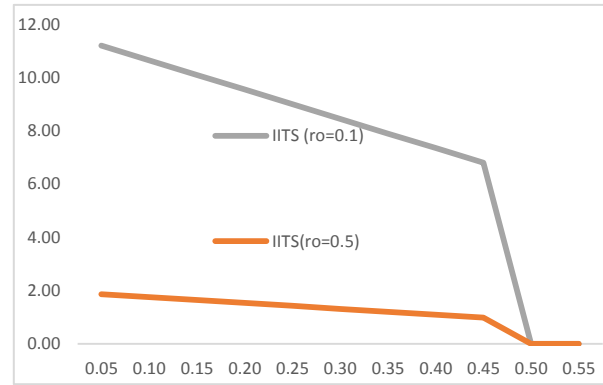

Figure 1(e) Sensitivity of PITS to ϖ and ρ(ro)
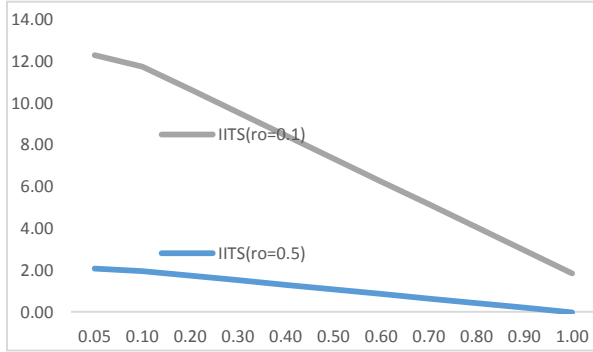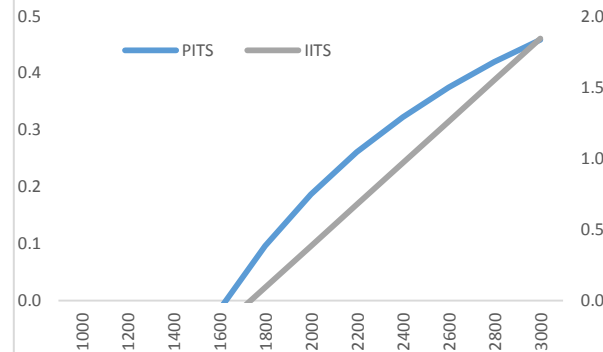

Figure 1(f) Sensitivity of IITS and PITS to q

Source: Authors' elaboration based on literature review

In **Figure 1**, by moving along the horizontal line from left to right, the value of the mentioned variables including $\rho, \frac{Q}{f}, \dots$ rises. The left vertical line and the blue line graph in the figure indicate the value of IITS, and the right vertical line and red line graph relate to PITS. As **Figure 1(a)** shows, by increasing ρ (the likelihood of the B period) and moving from left to right, IITS narrows but PITS widens. Moreover, with an increase in q (the output level), both IITS and PITS widen, but PITS starts to widen in the lower level of output. **Figures 1(b)**, **1(d)** and **1(e)** indicate that IITS narrows by a rise in $\frac{Q}{f}, \bar{\bar{\psi}}, \varpi$ (the value of the collateral and its related costs). Another point is that a rise in ρ intensifies the speed which IITS narrows. The last point is that an increase in the auditing costs sharply narrows PITS but an increase in ρ raises PITS for any rate for auditing costs.

CONCLUDING REMARKS

IBC and PLSC are two specific financial contracts with interesting features. Both of them are acceptable legally binding agreements that are frequently used in the financial market for investing purposes. However, their most important difference is how they share the inherently existent risks in the business activities between the parties involved. While IBC is known as a risk-shifting contract, PLSC is known as a risk-sharing one. Due to their specific natures, each of the parties' associated risk and return will be changed by the contract at least in the short term. Moreover, at the time of signing an investment contract, the state of the economy (SoE) that affects the participants' realised risk and return is a stochastic variable. By assuming the profit as an engine in decision-making and considering the fact that the contract and the role of the party may have an impact on the obtained profit, this research specifies eight profit functions to cover these issues. Specifying several payoff functions generates a structure that enables us to analyse the sensitivity of the player's decision-making to contracts and the player's role. Moreover, signing any contract needs all parties' consent. This means that there is full interaction between the participants. Then a mathematical GM is developed to reflect the interactivity of agents by a change in one factor.

The results showed that both PLSC and IBC are possible contracts in different SoE conditions. However, the possibility of PLSC is greater when the future is more likely to be a boom (B) period. The results of the theoretical model are verified by a numerical example which shows that PITS (IITS) widens (narrows) by a rise in the likelihood of B periods. Moreover, both IBC and PLSC are more feasible when the output level increases for any level of mentioned likelihood. Moreover, it showed that the parties' share of the revenue could be an important factor in improving PLSC.

Policymakers who are interested in improving PLSC in their economy to make it more resilient against crises may focus on the PLSC share (α) and provide a proper guideline for different SoE conditions.

The limitation of this study pertains to the setting aside of some important variables to simplify the model. For example, we know that the output is a direct function of the borrower's effort and must be explicitly considered in the model. Alternatively, the investor's effort (for monitoring, auditing, or advising the entrepreneur) may affect total output (either positively or negatively), but this model did not cater for this issue. Thus, future research may concentrate on comparing the resultant payoff of the two contracts to see how they create different payoffs and evolve over time in an interaction dynamic framework. Moreover, they may focus on a proper production function that reflects the lender's and borrower's effort.

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APPENDICES
Appendix A
A.1

$$E(\pi_b^{IBC}) = \rho(\pi_b^{IBCB}) + (1 - \rho)(\pi_b^{IBCNB})$$
$$= \rho[P^B.q^{IBC}(e^{IBC}, F) - c_b^{IBC}(e_b) - \bar{r}f - f - \bar{\psi}Q] + (1 - \rho)[P^{NB}.q^{IBC}(e^{IBC}, F) - c_b^{IBC}(e_b) - \bar{\psi}Q - \varpi Q] = \rho[P^B.q^{IBC}(e^{IBC}, F)] - \rho c_b^{IBC}(e_b) - \rho\bar{r}f - \rho f - \rho\bar{\psi}Q + (1 - \rho)[P^{NB}.q^{IBC}(e^{IBC}, F)] - c_b^{IBC}(e_b) - \bar{\psi}Q - \varpi Q + \rho c_b^{IBC}(e_b) + \rho\bar{\psi}Q + \rho\varpi Q$$
$$= q^{IBC}(e^{IBC}, F)[\rho P^B + (1 - \rho)P^{NB}] - (1 + \bar{r})\rho f - c_b^{IBC}(e_b) - (\bar{\psi} + \varpi - \rho\varpi)Q$$

A.2

$$E(\pi_l^{IBC}) = \rho(\pi_l^{IBCB}) + (1 - \rho)(\pi_l^{IBCNB}))$$

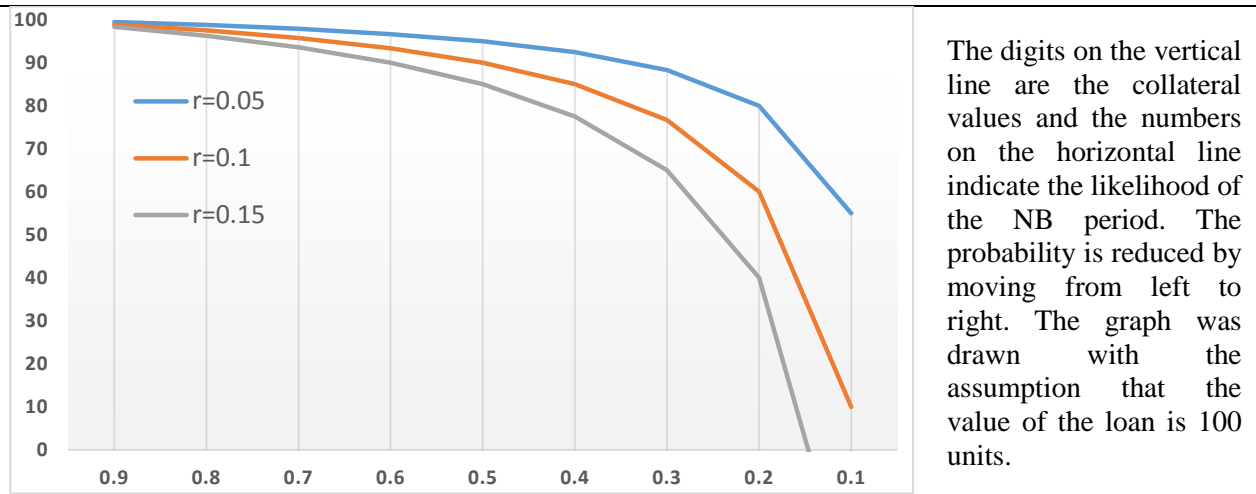Plugging for π_l^{IBCB} and $e(\pi_l^{IBCNB})$ form Equations (3) and (5) gives,

$$E(\pi_l^{IBC}) = \rho(\bar{r}f) + (1 - \rho)(Q - f)$$
$$= \rho\bar{r}f + (1 - \rho)Q - (1 - \rho)f$$
$$= (\rho\bar{r} + \rho - 1)f + (1 - \rho)Q$$

A.3

Figure 1.A illustrates how the value of the collateral is sensitive to the SoE and the Interest Rate.

Figure 1.A: The Sensitivity of the Required Collateral Value to the SoE and the Interest Rate

The digits on the vertical line are the collateral values and the numbers on the horizontal line indicate the likelihood of the NB period. The probability is reduced by moving from left to right. The graph was drawn with the assumption that the value of the loan is 100 units.

Source: Authors' own

Appendix B

$$E(\pi_b^{PLSC}) = \rho\pi_b^{PLSCB} + (1 - \rho)\pi_b^{PLSCNB}$$
$$= \rho\{\alpha[P^B q^{PLSC}(e(e_b), F)] - c_b^{PLSC}(e_b) - (F - f_1)\}$$
$$+ (1 - \rho)\{\alpha[P^{NB} q^{PLSC}(e(e_b), F)] - c_b^{PLSC}(e_b) - (F - f_1)\}$$
$$= \alpha\rho[P^B q^{PLSC}(e(e_b), F)] - \rho c_b^{PLSC}(e_b) - \rho(F - f_1) + \alpha[P^{NB} q^{PLSC}(e(e_b), F)] - c_b^{PLSC}(e_b)$$
$$- (F - f_1) - \alpha\rho[P^{NB} q^{PLSC}(e(e_b), F)] + \rho c_b^{PLSC}(e_b) + \rho(F - f_1)$$

Simplifying; $= \alpha[\rho P^B + (1 - \rho)P^{NB}]q^{PLSC}(e(e_b), F)] - c_b^{PLSC}(e_b) - (F - f_1)$

Appendix C
C.1

$E(\pi_l^{IBC}) = (\rho\bar{r} + \rho - 1)f + (1 - \rho)Q = 0 \rightarrow (\rho\bar{r} + \rho - 1) = \frac{(\rho-1)Q}{f} \rightarrow \rho\bar{r} = \frac{(\rho-1)Q}{f} + 1 -$

$\rho \rightarrow \rho\bar{r} = \frac{(\rho-1)Q-((\rho-1)f}{f} = \frac{(\rho-1)(Q-f)}{f} \rightarrow \bar{r} = \frac{\frac{(\rho-1)(Q-f)}{f}}{\rho} \quad \rightarrow \bar{r} = \frac{\rho(\rho-1)(Q-f)}{f} \rightarrow \bar{r} =$

$\frac{\rho(\rho-1)(Q-f)}{f}$

C.2

$E(\pi_b^{IBC}) = q^{IBC}[\rho P^B + (1 - \rho)P^{NB}] - [c_b^{IBC}(e_b) + (\bar{\psi} + \varpi - \rho\varpi)Q + (1 + \bar{r})\rho f] = 0$
$q^{IBC}[\rho P^B + (1 - \rho)P^{NB}] - c_b^{IBC}(e_b) - (\bar{\psi} + \varpi - \rho\varpi)Q = (1 + \bar{r})\rho f$
$q^{IBC}[\rho P^B + (1 - \rho)P^{NB}] - c_b^{IBC}(e_b) - (\bar{\psi} + \varpi - \rho\varpi)Q = \rho f + \bar{r}\rho f$
$\bar{r}\rho f = q^{IBC}[\rho P^B + (1 - \rho)P^{NB}] - c_b^{IBC}(e_b) - (\bar{\psi} + \varpi - \rho\varpi)Q - \rho f$
$\bar{r} = \frac{\{q^{IBC}[\rho P^B + (1-\rho)P^{NB}] - c_b^{IBC}(e_b) - (\bar{\psi}+\varpi-\rho\varpi)Q\}}{\rho f} - 1$

Appendix D
```
>> syms ro f Q q Pu Pd cb si om F M O
IITS=((q*(ro*Pu+(1-ro)*Pd)-cb-(si+om-ro*om)*Q)/(ro*f))-((ro*(ro-1)*(Q-f))/f)-1;
diff (IITS, ro)=(Q*om - q*(Pd - Pu))/(f*ro) - ((Q - f)*(ro - 1))/f - (ro*(Q - f))/f + (cb + Q*(om +
si - om*ro) - q*(Pu*ro - Pd*(ro - 1)))/(f*ro^2)
diff (IITS, Q)= - (ro*(ro - 1))/f - (om + si - om*ro)/(f*ro)
diff (IITS, f)=(ro*(ro - 1))/f + (cb + Q*(om + si - om*ro) - q*(Pu*ro - Pd*(ro - 1)))/(f^2*ro) +
(ro*(Q - f)*(ro - 1))/f^2
diff(IITS, si)=-Q/(f*ro)
diff(IITS,om)=(Q*(ro - 1))/(f*ro)

F=(Q*om - q*(Pd - Pu))/(f*ro) - ((Q - f)*(ro - 1))/f - (ro*(Q - f))/f + (cb + Q*(om + si - om*ro) -
q*(Pu*ro - Pd*(ro - 1)))/(f*ro^2);
diff(F,ro)= - (2*(Q - f))/f - (2*(Q*om - q*(Pd - Pu)))/(f*ro^2) - (2*(cb + Q*(om + si - om*ro) -
q*(Pu*ro - Pd*(ro - 1))))/(f*ro^3)

diff (IITS, f)=(ro*(ro - 1))/f + (cb + Q*(om + si - om*ro) - q*(Pu*ro - Pd*(ro - 1)))/(f^2*ro) +
(ro*(Q - f)*(ro - 1))/f^2
M=(ro*(ro - 1))/f + (cb + Q*(om + si - om*ro) - q*(Pu*ro - Pd*(ro - 1)))/(f^2*ro) + (ro*(Q -
f)*(ro - 1))/f^2
diff (M, f)= - (2*ro*(ro - 1))/f^2 - (2*(cb + Q*(om + si - om*ro) - q*(Pu*ro - Pd*(ro -
1))))/(f^3*ro) - (2*ro*(Q - f)*(ro - 1))/f^3
```

diff (IITS, Q)= - (ro*(ro - 1))/f - (om + si - om*ro)/(f*ro)
O= - (ro*(ro - 1))/f - (om + si - om*ro)/(f*ro)
diff(O,Q)= - (2*cb + 2*Q*(om + si - om*ro) - 2*q*(Pu*ro - Pd*(ro - 1)))/(Q^3*ro) - (2*ro*(ro - 1))/Q^2

Appendix E

syms cl cb F wa Pd Pu ro q S A B
PITS =1-((cl+cb+F*(wa+1))/(q*(ro*Pu+(1-ro)*Pd)));
diff(PITS,ro)= -((Pd - Pu)*(cb + cl + F*(wa + 1)))/(q*(Pu*ro - Pd*(ro - 1))^2)
diff(PITS,q)=(cb + cl + F*(wa + 1))/(q^2*(Pu*ro - Pd*(ro - 1)))
diff(PITS,F)= -(wa + 1)/(q*(Pu*ro - Pd*(ro - 1)))
diff(PITS,cl)=-1/(q*(Pu*ro - Pd*(ro - 1)))
diff(PITS,wa)= -F/(q*(Pu*ro - Pd*(ro - 1)))