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Article

Detailed view of a payout product of the old-age pension saving scheme in Slovakia

Detailed View of a Payout Product of the Old-Age Pension Saving Scheme in Slovakia[1]

Jana ŠPIRKOVÁ – Gábor SZŰCS** – Igor KOLLÁR**

Abstract

This paper discusses a selected product of the payout phase of the old-age pension saving scheme in Slovakia which is stated by Act 43/2004 Coll. on the Old-Age Pension Saving Scheme, in Article 46. It models and analyses the amount of pension annuities in the designed product according to mortality rates, euro area yield curves, specific composition of the insurance strain, expenses and other requirements of the authorities of the European Union and Act 39/2015 Coll. Additionally, the paper provides answers to current questions about the payout phase of pension savings. It points to all aspects relating to the determination of the amount of future pensions.

Keywords: *retirement age, old-age pension product, yield curve, annuity, risk loading*

JEL Classification: G22, G28

Introduction

Old-age pension saving represents the second pillar of the pension scheme based on contributions. This means that the amount of pension will depend on the contributions paid to the second pillar and returns from the contributions.

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[1] Igor Kollár and Jana Špirková have been supported by the Slovak Scientific Grant Agency VEGA No. 1/0093/17 *Identification of risk factors and their impact on products of the insurance and savings schemes*.

Gábor Szűcs has been supported by the Slovak Scientific Grant Agency VEGA No. 1/0251/16 *Quantitative analysis of models of interest rates in conditions of Eurozone and accession countries and its applications*; and by the VEGA No. 2/0054/18 *New statistical methods for specific classes of probability distributions and their applications*.

The area of old-age pension saving in Slovakia is regulated by Act 43/2004 Coll. on the Old-Age Pension Saving Scheme and on amendments and supplements to certain laws (hereinafter referred to as Act), with status as of January 1, 2019.[2]

The first pensions from the old-age pension saving system were paid out on January 1, 2015. One of the benefits for the savers is that they can buy an annuity in a life insurance company with a licence to pay out pensions from old-age pension savings.

Moreover, the savers are not obliged to buy a life annuity, but they can repurpose their savings and leave them as an inheritance as is declared in Article 40 of the Act. The Social Insurance Agency has established a so-called Central Information Offer Register (in Slovak Centrálny informačný ponukový systém – CIPS, hereinafter referred to as Register) to mediate relevant offers for old-age pension saving products to the savers. This Register will guarantee transparency of the system and reduce the administrative burden of the savers, who will get all offers in one place.

The paper provides answers to current questions about the payout phase of old-age pension saving. It points to all aspects relating to the determination of the amount of future pensions. The paper also discusses so-called effective premium, which represents the amount which will be paid in the form of monthly pensions after taking into account risk loadings, costs and expenses. These questions resonate not only with the younger generation, but especially with the generation approaching the retirement age.

Based on the data provided by Social Insurance Agency in Slovakia, the average amount of the pension in 2018 reached 24.16 euros. This value is not indicative in our modelling because the average accumulated sum is unknown. However, estimates of monthly pension annuities for 62 year`s old pensioner are published in the Register by three insurance companies in Slovakia (Bidding list, 2019). After recalculation of accumulated sum to 10,000 euros, monthly pension are following: 29.89 euros, 32.62 euros and 36.86 euros.

The aim of our paper is to model and analyse the selected product of the payout phase of the old-age pension scheme, namely Product 1 which is listed in Article 46 of the Act, as an old-age annuity or an early retirement annuity which does not include raising of the pension and does not include survivors' benefits. Our topic is specific in that we and the public have only a limited access to the real insurance companies' methodologies by which actuaries calculate the amount of the annuities in the old-age pension saving. This paper provides insight into what is happening with the savings accumulated by the savers after their relocation to the payout phase, i.e., after buying life pension annuity in a life insurance

[2] Available at: <https://www.nbs.sk/_img/Documents/_Legislativa/_BasicActs/A43-2004.pdf>.

company. The paper, among other things, provides insights into insurance companies' calculations and methodologies that allow them to calculate the amount of monthly annuities in old-age pension saving. Such information may be valuable to both the savers and the professional public.

While adhering to the requirements and recommendations of all applicable laws and directives of the Slovak Republic and European Union, we provide answers to the following research questions.

Q1: *What is the effective premium, i.e., how much of the accumulated 10,000 euros will be used to pay a life annuity?*

Q2: *Considering all modelled risk loadings, what is the lowest monthly pension annuity we should expect from the accumulated sum of 10,000 euros?*

Q3: *For how many years should a pensioner receive a monthly pension annuity to get back at least 10,000 euros?*

Q4: *To what extent does the requirement of a 7-year annuity (84 monthly annuities) in the Product 1 affects the amount of the monthly annuity in comparison with a product that does not include this requirement, i.e., versus classical monthly pension annuity?*

Moreover, we want to take a position on the following hypotheses.

H1: *Life insurance companies consume more than 25% of pensioner savings, which means that the whole life annuity paid out is less than 75% of the savings.*

H2: *Actual life expectancy for 62 year's old pensioner is 19.49 years. In the case that 62 years old person lives for another 20 years, the whole saved amount will return to the person in the form of monthly pension payments.*

For product analysis we used mortality tables (Mortality Tables, 2017; The Human Mortality Database, 2017) and their adjustment according to the Lee-Carter model of longevity. Our model also includes the returns from investment funds which are modelled by the Svensson yield curve whose parameters are published on a daily basis by the European Central Bank (ECB), (ECB, 2017a). Moreover, it meets requirements of the Council Directive 2004/113/EC of 13 December 2004 implementing the principle of equal treatment between men and women in the access to and supply of goods and services, Directive 2009/ 138/EC of the European Parliament and of the Council of 25 November 2009 on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II) and the European Insurance and Occupational Pensions Authority and the European Securities and Markets Authority.

All calculations were made by the MS Office Excel 2016 system and the R software with demography package, (R Core Team, 2016; Hyndman et al., 2014). Moreover, we were inspired by psychological assessment of this problem in Bačová and Kostovičová (2018), and by the scientific research of authors Albrecher et al. (2016).

The remainder of this paper is structured as follows. Section 1. – *Preliminaries* gives basic actuarial concepts and notations as building blocks of actuarial modelling. Section 2. – *The Offer of Life Old-age Pension Stated by the Article 46* describes actuarial model of the Product 1. Section 3. – *Evaluation of the Product 1* contains analysis of the Product 1 and relevant discussion. Finally, *Conclusion* presents our main results, answers to research questions and attitudes to established hypotheses. Moreover, this section also contains our future plans to model and analyse other products that flow from the second pillar pension saving.

1. Preliminaries

In this section we present basic actuarial notations, assumptions and requirements that are applied to modelling of not only pension insurance products but also to modelling of all life insurance products, in general. We also mention risk-free interest rate structure, model of longevity, administrative and collection costs related to pension saving products.

1.1. Survival and Mortality Probabilities

The basic building blocks in modelling of all life insurance products are the relevant survival and mortality probabilities which are given as follows:

${}_t p_x$ – the probability that individual at age *x* survives to at least age $x+t$,

${}_t q_x$ – the probability that individual at age *x* dies before age $x+t$,

${}_{r|t} q_x$ – the probability that individual at age *x* survives *r* years, and then dies in the subsequent *t* years, that is, between ages $x+r$ and $x+r+t$.

The probability ${}_{r|t} q_x$ is also called a deferred mortality probability, because it is the probability that death occurs in an interval following a deferred period. It can be calculated by formula

$$ {}_{r|t} q_x = {}_r p_x - {}_{r+t} p_x \tag{1} $$

Because pensions from the second pillar pension savings are paid monthly, in advance, we consider fractional age assumption using linear interpolation. Linear interpolation between integer ages and the uniform distribution of deaths assumption is the most common fractional age assumption, (Dickson et al., 2009). It can be formulated as follows.

Definition 1 (Dickson et al., 2009)

For integer x, provided the uniform distribution of deaths, and for $0 \le s < 1$, assume that

$$ {}_s q_x = s \times q_x \tag{2} $$

1.2. Yield Curves

We can see interesting approaches to modelling the effect of the interest rate on the level of pensions in literature. Very interesting approach can be found in Mihalechová and Bilíková (2015) and Melicherčík, Szűcs and Vilček (2015).

In our research we use the yield curve of the ECB, because insurance companies invest premiums mainly in bonds issued by euro area countries. Moreover, we decided to use the Svensson yield curve because it is available online on the ECB's website (ECB, 2017a; Svensson, 1994).

A yield curve is a representation of the relationship between market remuneration rates and the remaining time to maturity of debt securities. It can also be described as the term structure of interest rates. The euro area yield curve shows separately AAA-rated euro area central government bonds and all euro area central government bonds (including AAA-rated). It is updated every TARGET business day at noon, 12:00 Central European Time, (ECB, 2017a).

In this part, we give basic formula of the Svensson yield curve according to ECB (2017b). For more information see Svensson (1994) and Aljinović, Poklepović and Katalinić (2012).

Definition 2 *The Svensson yield curve* is given by

$$R(z) = \beta_0 + \beta_1 \times \frac{1 - \exp\left(-\frac{z}{\tau_1}\right)}{\frac{z}{\tau_1}} + \beta_2 \times \left[\frac{1 - \exp\left(-\frac{z}{\tau_1}\right)}{\frac{z}{\tau_1}} - \exp\left(-\frac{z}{\tau_1}\right)\right] + \\ + \beta_3 \times \left[\frac{1 - \exp\left(-\frac{z}{\tau_2}\right)}{\frac{z}{\tau_2}} - \exp\left(-\frac{z}{\tau_2}\right)\right] \tag{3}$$

where

$R(z)$ – yield from a bond investment with continuous compounding (% p.a.),

z – term to maturity, $z \in \left]0,\ T_{max}\right]$,

T_{max} – maximum term to maturity,

$\beta_0,\ \beta_1,\ \beta_2,\ \beta_3,\ \tau_1,\ \tau_2$ – parameters of the Svensson yield curve, where $\beta_0,\ \tau_1,\ \tau_2$ must be positive.

1.3. Lee-Carter Model of Longevity

An integral part of our model is the modelling of longevity as an essential part of the Solvency Capital Requirement.

Lee-Carter model is one of the models currently used by commercial insurance companies, (Lee and Carter, 1992). It is defined by

$$m_{x,t} = \exp\left(a_x + b_x \times k_t + \epsilon_{x,t}\right),\ x \in \mathrm{X},\ t \in \mathcal{T} \tag{4}$$

where

$m_{x,t}$ – the central mortality rate of a person at age x in year t,

a_x – age-specific model parameter that does not depend on time,

b_x – age-specific model parameter that represents how rapidly or slowly mortality at each age varies when the general level of mortality changes,

k_t – time-varying index (independent of age), reflecting the general level of mortality,

$\epsilon_{x,t}$ – random noise (error member) with zero mean value and dispersion σ^2 for $x \in \mathrm{X},\ t \in \mathcal{T}$, where X is the set of ages and $\mathcal{T}$ is the set of times.

Estimating the Lee-Carter model parameters and predicting future mortality rates are performed in the R software using the demography package (R Core Team, 2016; Hyndman et al., 2014), see Figure 1.

1.4. Administrative and Collection Costs

In our model, we take into account The Act No. 39/2015 Coll. on insurance, Part Three,[3] Article 23 General governance requirements, paragraph (3), which declares that insurance companies shall perform their activities in a prudent manner. Moreover, under this Act, Article 69 – Premiums under new contracts, insurance companies may take account of all aspects of their financial situation.

In our modelling we assume a new starting insurance company and therefore we use costs that are published in actuarial literature so that we comply with the requirements of a prudent business. We especially consider initial and administrative costs and also insurer's legitimate expenditures. More information can be found in Subsection 2.2. – *Basic Concepts of Pension Annuity Modelling*.

2. The Offer of Life Old-age Pension Stated by the Article 46

In this part we focus on modelling and analysis of the life old-age pension and life early retirement pension pursuant to Article 46 of the Act.

In particular, we concentrate on the Product 1 listed in paragraph (1), "On the issuance date of the certificate, the insurer shall, through the offer system: (a) draw up for the saver an offer of an old-age annuity or an early retirement annuity which: 1. does not include raising of the pension and does not include survivors' benefits".

[3] Part Three – Requirements for the performance of insurance and reinsurance activities, Title one – System of governance.

2.1. Modelling of the Product 1

Article 46 says that an insurer will draw up an offer of old-age annuity or early retirement annuity for the saver on the date of issue of the certificate through the Register. The selected Product 1 for the saver is as follows: product includes permanent monthly annuity and the payment of a lump sum equal to not yet paid monthly annuities in the case of the beneficiary death during the period of the first seven years of pension payment, (does not include raising of the pension and does not include survivors' benefits). In case the beneficiary of whole life old-age annuities dies during the first seven years of payment, the rest of the sum to be paid during this period will be paid to a person designated by the saver in the old-age pension agreement. If there is no such person, the amount designated for the payment shall become the subject of inheritance proceedings.

2.2. Basic Concepts of Pension Annuity Modelling

Pension annuity contracts offer a regular series of payments. If the pension annuity continues until the death of the pensioner, it is called a whole life annuity. The buying of a whole life annuity guarantees that the pension income will not run out before the pensioner dies.

2.2.1. Basic Notations

Firstly, we introduce basic notations which are used in formulas of individual products:

- S – accumulated sum, single premium in monetary units (euros),
- $P(z)$ – discounting factor; $P(z) = \exp\left(\dfrac{-R(z)}{100\ \%} \times z\right)$,
- x – retirement age, in years,
- ω – maximum age to which a person can live to see,
- α – initial costs as a % from the first yearly annuity,
- β – administrative costs as a % p.m. from technical provisions on whole life annuity in advance,
- δ_1 – the insurer's legitimate expenditures as a % from the amount of the lump-sum premium related to the payment of this amount in cash or its transfer to a non-euro area country in a case of client's death,
- δ_2 – the insurer's legitimate expenditures as a % from the amount of the expected present value of the sum of not yet paid monthly annuities in the case of the beneficiary death during the period of the first seven years of pension payment,
- IC – initial costs as an absolute amount in monetary units independent on an accumulated sum.

2.2.2. Actuarial Modelling of Product 1

As we have mentioned in Subsection 2.1., Product 1 includes permanent monthly annuity and the payment of a lump sum equal to not yet paid monthly annuities in the case of the pensioner's death during the period of the first seven years of pension payment.

The equivalence equation with financial flows at the time of closing the policy agreement is as follows:

$$\begin{aligned} S \;=\; & 12\times MP_x \times a_x^{(12)} + A^1_{x:1/12}\times S\times\left(1-\frac{\delta_1}{100\%}\right)+A^1_{x:1/12}\times S\times\frac{\delta_1}{100\%}+ \\ & +\;12\times MP_x\times\left(MA^{(12)}\right)^1_{x:7}\times\left(1-\frac{\delta_2}{100\%}\right)+ \\ & +\;12\times MP_x\times\left(MA^{(12)}\right)^1_{x:7}\times\frac{\delta_2}{100\%}+ \\ & +\;12\times MP_x\times\frac{\alpha}{100\%}+IC+12\times MP_x\times\frac{\beta}{100\%}\times\sum_{t=1}^{12(\omega-x)}{}_{\frac{t}{12}|}a_x^{(12)} \end{aligned} \tag{5}$$

On the left-hand side is an accumulated sum S which represents a single premium of this product.

On the right-hand side are following financial flows, sequentially:

1) The value $12\times MP_x\times a_x^{(12)}$ is the expected present value of whole life monthly annuity MP_x in advance, where

$$a_x^{(12)}=\sum_{t=1}^{12(\omega-x)}\frac{1}{12}\times{}_{\frac{t}{12}}p_x\times P\left(\frac{t}{12}\right)$$

is the expected present value of whole life benefits in advance of $\frac{1}{12}$ of monetary units (m.u.), 12 $\times$ per year, under condition that the client is alive. This formula expresses requirement under the Act, Article 32 – Annuity, paragraph (1), where is listed, "Old-age annuities and early retirement annuities shall be paid by the insurer to the beneficiaries of these annuities until their death".

2) The second expression $A^1_{x:1/12}\times S\times\left(1-\frac{\delta_1}{100\ \%}\right)$,

where $$A^1_{x:1/12}={}_{\frac{1}{12}}q_x\times P\left(\frac{1}{12}\right)$$

is the expected present value of benefit in the amount of 1 m.u. in the case of pensioner's death during the first month of retirement. It represents the situation under the Act, Article 46g, paragraph (5).

The value $A^{1}_{x:1/12} \times S \times \dfrac{\delta_1}{100\ \%}$ expresses expenditures of the insurance company associated with a quick withdrawal of finances from investment funds with respect to Article 46g.

In this case we assume the insurer's legitimate expenditures are just equal to the insurer's costs associated with a quick withdrawal of finances from investment funds.

3) The expression $12 \times MP_x \times (MA^{(12)})^{1}_{x:7} \times \left(1 - \dfrac{\delta_2}{100\ \%}\right)$

where

$$\left(MA^{(12)}\right)^{1}_{x:7} = \sum_{t=1}^{83} \frac{84-t}{12} \times {}_{\frac{t}{12}|\frac{1}{12}}q_x \times P\left(\frac{t+1}{12}\right)$$

represents the expected present value of the sum of not yet paid monthly annuities in the amount of $\dfrac{1}{12}$ of m.u. in the case of the pensioner's death during the period of the first seven years of pension payment, which relates to expenditures which are described in the Act, Article 32.

The expression $12 \times MP_x \times (MA^{(12)})^{1}_{x:7} \times \dfrac{\delta_2}{100\ \%}$ gives expenditures of the insurance company associated with a quick withdrawal of finances from investment funds. Once again we assume the insurer's legitimate expenditures are just equal to the insurer's expenditures associated with a quick withdrawal of finances from investment funds.

4) The expression $12 \times MP_x \times \dfrac{\alpha}{100\ \%}$ gives lump sum initial costs from the first yearly annuity.

5) Initial costs are denoted by IC as an absolute amount in monetary units.

6) The expression

$$12 \times MP_x \times \frac{\beta}{100\ \%} \times \sum_{t=1}^{12(\omega-x)} {}_{\frac{t}{12}|}a_x^{(12)}$$

is the expected present value of administrative costs, where the expected present value of deferred annuities for $t = 1,\ 2,\ \ldots,\ 12(\omega - x)$, is given by

$${}_{\frac{t}{12}|}a_x^{(12)} = \frac{1}{12} \times \sum_{r=t}^{12(\omega-x)} {}_{\frac{r+1}{12}}p_x \times P\left(\frac{r+1}{12}\right)$$

hence total sum of provisions is as follows $\sum_{t=1}^{12(\omega-x)} {}_{\frac{t}{12}|}a_x^{(12)}$.

Directly, from described equivalence equation (5) we get formula on calculation of monthly pension annuity MP_x of this product. It is given by

$$MP_x = \frac{S \times \left(1 - A^{1}_{x:1/12}\right) - IC}{12 \times \left(a_x^{(12)} + \frac{\alpha}{100\ \%} + (MA^{(12)})^{1}_{x:7} + \frac{\beta}{100\ \%} \times \sum_{t=1}^{12(\omega - x)} {}_{\frac{t}{12}}| \, a_x^{(12)} \right)} \tag{6}$$

Remark 1. In our model we could consider expenditures of the insurance company associated with a quick withdrawal of finances from investment funds other than the insurer's legitimate expenditures related to the payment of the amount in cash or its transfer to a non-euro area country. These expenditures do not affect monthly annuity but only on the amount of lump sum which will be paid out in the case of client's death.

3. Evaluation of the Product 1

As the basic building blocks of our calculations are as follows:

- Mortality tables for the year 2016 for total population which are published on the web page of the Statistical Office of the Slovak Republic, (Mortality Tables, 2017), see Figure 1.
- The Lee-Carter model of longevity and lower boundary of the 90% prediction interval for the Lee-Carter model, 1950 – 2014 and predictions on 2015 – 2063, total population (The Human Mortality Database, 2017), see Figure 1.
- Linear interpolation between integer ages and the uniform distribution of deaths assumption.
- The Svensson yield curve on 7 December 2017, for AAA rated bonds and all bonds (European Central Bank, 2017), see Figure 2.
- Initial costs, administrative costs and insurer's legitimate expenditures.

Annual probabilities of death of an individual at age x acquired from the Statistical Office of the Slovak Republic and the Human Mortality Database modified using the Lee-Carter Model of Longevity are illustrated on Figure 1, as well as lower bound of the Lee-Carter Model of Longevity. This lower bound expresses risk loading of longevity.

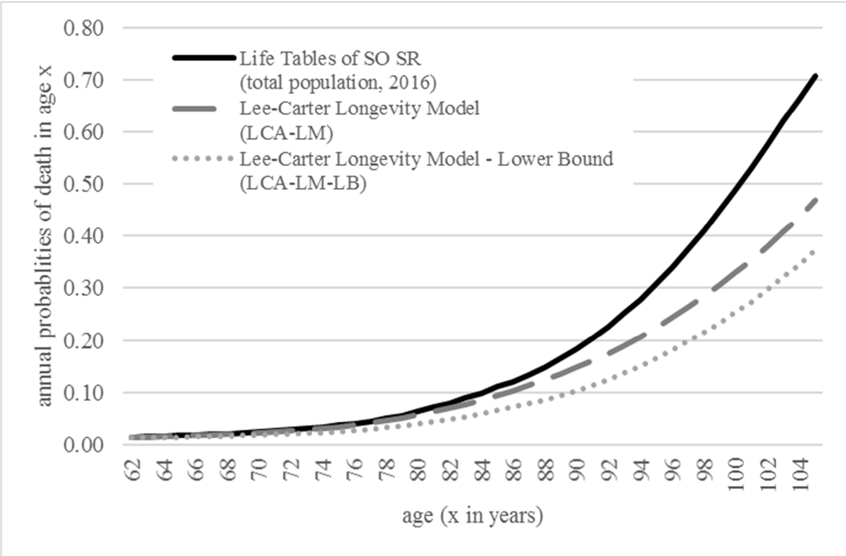
Figure 2 illustrates risk-free yield curves which are published by ECB. From this figure it is possible to see negative yields for bonds with term to maturity approximately up to 5 years for all bonds and 7 years for AAA bonds.

In our paper we offer a general model of pension calculation for x-aged individual, but as a model situation we offer analysis for a pensioner aged 62 years.[4]

Remark 2. We do not take into account Article 42a of the Act 43/2004 Coll. – Share in the income surplus from the investment of technical reserves, paragraph (1).

F i g u r e 1

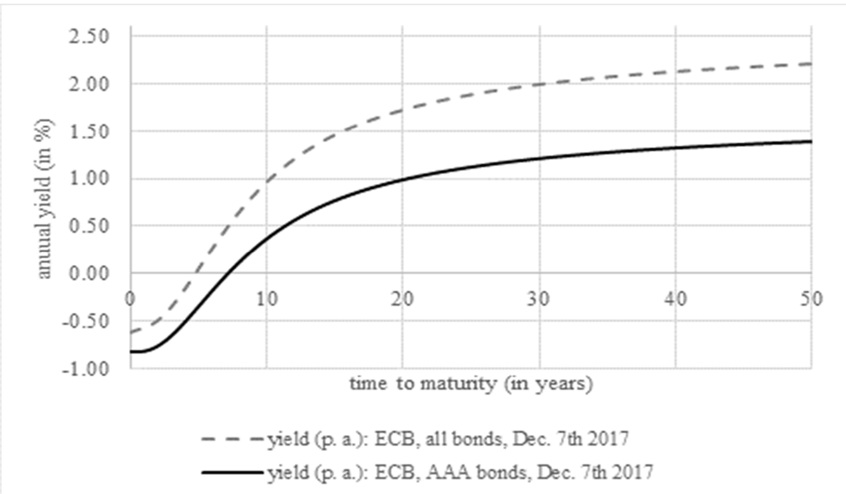
Life Tables with Respect to the Statistical Office of the Slovak Republic and the Human Mortality Database – Lee-Carter Model of Longevity



Source: Authors, Mortality Tables (2017), The Human Mortality Database (2017).

F i g u r e 2

Svensson Yield Curves of the European Central Bank



Source: Authors; ECB (2017).

[4] Ministry of Labor, Social Affairs and Family of the Slovak Republic pursuant to § 65a par. 3 of Act No. 461/2003 Coll. on Social Insurance as amended by Act No. 252/2012 Coll. Provides in § 2: The retirement age for the calendar year 2018 is 62 years and 139 days. For more information, see <http://www.socpoist.sk/starobny-dochodok/1286s>.

3.1. Loading Assessment

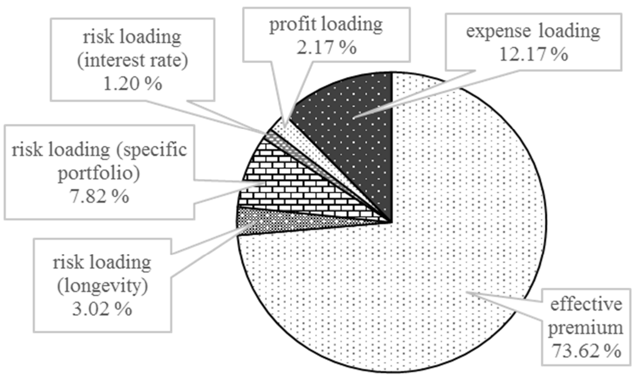
All loadings are built into the insurance costs. These amounts cover the operating costs of the insurance company. For determination of the correct amount of the monthly pension annuity we start using formula (5) without considering the costs of our virtual insurance company and longevity, i.e., we use the basic life tables published by the Statistical Office of the Slovak Republic (SO SR) for the total population from 2016 (Mortality Tables, 2017). We assume that an insurance company has just these tables approved by the National Bank of Slovakia. Moreover, we assume risk free interest rates published by the ECB (2017a) as "all bonds" and the accumulated sum as a single premium 10,000 euros. We receive the Monthly Pension annuity MP_{62} = 48.62 euros.

3.1.1. Calculation of Loading Components

In this part we offer the process of determining the loadings related to pension annuity. Specifically, we focus on longevity, specific composition of the insurance strain, market risk of the interest rate, profit of an insurance company and expense loadings. Percentage distribution of the effective premium and individual loadings for 62 year old person with the accumulated sum S = 10,000 euros is illustrated on Figure 3.

F i g u r e 3

Percentage Distribution of Effective Premium and Loadings for 62-aged Person with the Accumulated Sum 10,000 Euros



Source: Authors.

I. Calculation of the first component of the risk loading – longevity

If we use the same assumptions as in the previous case, but instead of life tables of the SO SR we use the Lee-Carter Longevity Model (LCA-LM), we obtain the Monthly Pension annuity with the risk loading 1 as follows: MP_{62} (risk loading 1) = 47.16 euros.

II. Calculation of the second component of the risk loading – specific composition of the insurance strain

Insurance companies paying pensions from old-age pension savings say the following: A person who buys life monthly annuity will live longer than a regular member of the population. Savers in pension savings have earned more than average over their active years. Savers in pension savings have access to better health care and therefore they will live longer.

Additionally, the anti-selection principle may work, which means that seriously ill pensioners do not buy a whole life pension, as a consequence, natural balance does not exist among those to whom the pensions will be paid only for a short period of time, and those to whom the pensions will be paid for a very long time. Under the same assumptions as in the previous case but using the Lower Bound of the 90% prediction interval for the Lee-Carter Longevity Model (LCA-LM-LB), we obtain the Monthly Pension annuity with the risk loading 2 as follows: MP_{62} (risk loading 2) = 43.35 euros.

III. Calculation of the third component of the risk loading – market risk of the interest rate

Applying the same assumptions as in the previous case, but considering yield curve of AAA rated bonds, the Monthly Pension annuity with the risk loading 3 is MP_{62} (risk loading 3) = 39.92 euros.

IV. Calculation of the fourth component of the loading – profit loading

Profit loading is defined as follows: the insurer will account for half of the α costs and the initial *IC* costs, and both costs are charged at the time of the life insurance contract signing. The insurance company will satisfy the shareholders of the joint-stock company. Hence, the insurance company expects to be able to invest only in AAA rated bonds considering extra longevity, i.e., considering the Lower Bound of the 90% prediction interval for the Lee-Carter Longevity Model (LCA-LM-LB). Moreover, we apply illustrative α = 20% from the first yearly annuity (half of this goes into the profit loading, half is used to cover the initial costs of the insurance company). Moreover, we add the initial costs *IC* = 300 euros (of which 200 euros goes into a profit loading and 100 euros goes to the agent as a commission). Those 100 euros will be counted in the next step as an expense loading. If we use the same assumptions as in the previous case and if we also add the costs above, the Monthly Pension annuity with the loading 4 is MP_{62} (profit loading 4) = 38.93 euros.

V. Calculation of the fifth component of the loading – expense loading

For determination of the expense loading, we assume illustrative administrative costs β = 0.1%, half of α costs, i.e., 10% and 100 euros from initial costs *IC*. Next, we assume yields of AAA rated bonds, considering the above mentioned

Model LCA-LM-LB. The Monthly Pension annuity with respect to our assumptions with loading 5 is as follows MP_{62} (expense loading 5) = 33.41 euros. Based on the above mentioned risk loads we can calculate Effective Premium. For 62 year old, Effective Priemium is 73.62%.

Effective Premium as well as the share of individual risk loadings varies depending on the retirement age. Table 1 offers the amount of the monthly pension annuity and distribution of the effective premium and risk loadings or profit and expense loadings, for retirement ages 62, 65, 70, 75 and 80 years.

Table 1

Monthly Pension Annuities and Distribution of Effective Premium and Risk Loadings According to Retirement Age with the Accumulated Sum 10,000 Euros

Retirement Age x (years)	**62**	**65**	**70**	**75**	**80**
Monthly Pension (euros)	*33.41*	*36.45*	*42.98*	*52.03*	*63.96*
Effective premium (%)	73.62	72.98	72.00	71.34	71.89
Risk loading 1 (longevity) (%)	3.02	3.52	4.37	5.40	6.27
Risk loading 2 (specif. Portfolio) (%)	7.82	8.93	10.64	11.78	11.70
Risk loading 3 (interest rate) (%)	1.20	1.21	1.19	1.13	1.04
Profit loading 4 (%)	2.17	2.16	2.17	2.21	2.31
Expense loading 5 (%)	12.17	11.20	9.63	8.14	6.79

Source: Authors.

From Table 1 and Figure 3 it is obvious that effective premium as a net present expected amount of monthly annuities is 7,362 euros (from accumulated sum 10,000 euros), risk of longevity is 302 euros, risk of the specific portfolio is 782 euros, risk of the interest rate is 120 euros, profit loading is 217 euros and expected present value of all costs and expenses is at 1,217 euros. This last value might seem too high for someone, but insurance companies do not have just ordinary basic administrative costs, but also high initial costs associated with starting pension payment services.

Additionally, it is very interesting to monitor the development of individual loadings on the basis of increasing retirement age. From Table 1 it directly follows that effective premium is the highest for 62 year old person.

Risk loadings of longevity and specific portfolio increase, expense loading decreases significantly with the increasing retirement age and risk loading of the interest rate and profit loading do not change significantly.

3.1.2. Annuity without a 7-year Guaranteed Pension and Lower Bound of the Pension Annuity

Oftentimes the question of whether a 7-year guaranteed pension (84 payments) influences the amount of monthly annuity arises and if so to what extent. Also, economists are very interested in the level of the lower bound of the monthly annuity. We assume illustrative administrative costs β = 0.1%, half of α costs, i.e., 10% and 100 euros from initial costs *IC*, the Model LCA-LM-LB (see paragraph *V. Calculation of the fifth component of the loading – expense loading*) and zero interest rate instead of AAA rated bonds, i.e., zero yields.

In Table 2, monthly pension annuities, monthly pension annuities without counting the risk of paying 84 pensions, lower bound of the amount of the pension annuities and lower bound of the amount of the pension annuities without counting the risk of paying 84 pensions are given based on the retirement age.

Table 2

The Amount of the Monthly Pension Annuities w.r.t. (6), According to Retirement Age with the Accumulated Sum 10,000 Euros and Zero Yields

Retirement Age x (years)	**62**	**65**	**70**	**75**	**80**
Monthly Pension (euros)	33.41	36.45	42.98	52.03	63.96
Monthly Pension without counting the risk of paying 84 pensions (euros)	33.88	37.09	44.11	54.45	69.94
Lower Bound of Monthly Pension (euros)	29.73	33.03	40.06	49.76	62.53
Lower Bound of Monthly Pension without counting the risk of paying 84 pensions (euros)	30.10	33.54	41.03	51.94	68.15

Source: Authors.

If we did not take a 7-year guaranteed pension into account, then the pension for a 62 year old would be just 0.47 euros higher. However, for an 80 year old the difference would be 5.98 euros. As people generally start retiring from the age of 62 to 70, we can say that incorporating a 7-year guaranteed pension (84 payments) is appropriate because the amount of the pension will not be greatly affected.

Conclusion

In our paper, we stated the amount of monthly pension annuity of the old-age pension which is stated by Act 43/2004 Coll. on the Old-Age Pension Saving Scheme and by amendments and supplements to certain laws.

We analysed annuity which is given by Article 46 of the Act as Product 1. By applying Directive 2009/138/EC of the European Parliament and of the Council, we estimated the probability-weighted average of future cash-flows, taking account of the time-value of money, using the Svensson yield curve of the December 7, 2017. Moreover, we estimated risk loadings related to longevity, specific portfolio, interest rate, administrative expenses and also profit of the insurance company.

Now, we offer our answers to research questions and take a stance to established hypotheses which we have set out in the introduction to the paper.

To Q1: The expected effective value of the accumulated amount of 10,000 euros is 73.62%, i.e., 7,362.00 euros.

To Q2: From the accumulated sum of 10,000 euros we should expect the lowest monthly pension annuity for 62-age-old pensioner in the amount of 29.73 euros. For more information see Table 2.

To Q3: If the monthly pension annuity for a 62-year-old pensioner would be in the amount 33.41 euros (see Table 1), the pensioner would have to be alive for at least 25 years to return the invested 10,000 euros. If the monthly pension annuity would be in the amount 29.73 euros (see Table 2), the pensioner would have to be alive for at least 28 years.

To Q4: If we assume whole life monthly pension annuity, without any further specification during the period of the first seven years of pension payment, as is described in Subsection 3.1, monthly pension annuity could be 0.47 euros higher for a 62 year old and 5.98 euros higher an 80 year old.

To H1: From Figure 1 and also Table 1 it is obvious that effective premium represents only 73.62% of the accumulated sum, and hence we accept the assumption of the hypothesis H1.

To H2: If we consider a 62-year-old pensioner and monthly pension annuity in the amount of 33.41 euros, in order to get the 10,000 euros investment back, he/she would have to get a pension for at least 25 years, so in this case we would reject the hypothesis H2.

In the future we plan to continue in our detailed investigation of other products which are stated by the Act.

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