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# An Equilibrium Model for the Romanian Economy 

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#### Abstract

The model presented in this article is an adaptation of the IS-LM model for an open economy in which both the static aspects and dynamic ones are approached. The determination of marginal main indicators of GDP and interest rates, allow to identify problems and the directions of action to achieve economic equilibrium.


Keywords: equilibrium, GDP, investments, interest rate, consumption

## 1 Introduction

The economic equilibrium problem, has origins and manifestations lost in the mists of time.
Economic thinkers from different current and ideologies as François Quesnay, Léon Walras, Vilfredo Pareto, Alfred Marshall studied this problem.

In the XX century, John Maynard Keynes formulate a first economic equilibrium model for a closed economy without governmental sector.
Because the equilibrium problem bore controversies on economics, it get further researches, today being analyzed the fluctuations that accompany this process.
Within theory of economic equilibrium, a synthetic analysis it is the IS-LM model consisting of simultaneous equilibrium in two markets, money market and the goods and services in an autarkic economy.
Starting from Keynesian macroeconomic equilibrium, in 1937, Roy Harrod, James Meade and John Hicks tried to express mathematical majors relations of Keynes' theory (Hahn, F.H., 1977).

Subsequent developments of Alvin Hansen of 1949 and 1953 play an important role in systematizing known IS-LM model, in his book (Hansen A.H., 1959) in order to get the curve IS, Hansen calls the investment demand function of Keynes and the neoclassical paradigm and for the LL curve is the curve of points where supply and demand (Beaud M., Dostaler G., 1996).

The IS-LM model (King R. G., 1993; Lawn P. A., 2003; Martínez-García E., Vilán D., 2012; Romer D., 2000; Schmitt-Grohe S., Uribe M., 2002; Weerapana, A., 2003) was the basis for further researches - theoretical or empirical.
After Samuelson and Solow which include the original model of the Phillips curve (1960), Fleming Mundell and Fleming include balance of payments (1960 and 1962).

Also, Modigliani and Friedman use the consumption function (1954 and 1957) and Tobin includes the demand for money (1958).

[^0]Although economic literature that explores New Open Economy Macroeconomics (NOEM models) is not as rich as that of the closed economy model, it is a significant theoretical edifice for the current macroeconomic modeling: Bergin (Bergin P., 2004), Schmitt-Grohe and Uribe (Smith, R.P.; Zoega, G., 2009), Justiniano and Preston (Justiniano A., Preston B., 2008), (Justiniano A., Preston B., 2010), Martínez-García and Vilán (Martínez-García E., Vilán D., 2012).

The new approach enables researchers to explain the new changes that have occurred in the international macroeconomic environment.

In this paper we propose, based on ideological vision and studies of the most important researchers in the field to determine a model for an open economy, with applications on the Romanian case.

## 2 The model equations

The first equation of the model is the formula of the aggregate demand:
(1) $\mathrm{D}(\mathrm{t})=\mathrm{C}(\mathrm{t})+\mathrm{G}(\mathrm{t})+\mathrm{I}(\mathrm{t})+\mathrm{EX}(\mathrm{t})-\mathrm{IM}(\mathrm{t})$
where

- $\quad \mathrm{D}(\mathrm{t})$ - the aggregate demand at the moment t ;
- $\mathrm{C}(\mathrm{t})$ - the actual final consumption of households at the moment t ;
- $G(t)$ - the actual final consumption of the government at the moment $t$;
- $\mathrm{I}(\mathrm{t})$ - the investments at the moment t ;
- EX(t) - the exports at the moment $t$;
- $\quad \operatorname{IM}(\mathrm{t})$ - the imports at the moment t

A second equation relates the actual final consumption of households according to disposable income:
(2) $\mathrm{C}(\mathrm{t})=\mathrm{C}_{\mathrm{V}} \mathrm{DI}(\mathrm{t})+\mathrm{C}_{0}, \mathrm{C}_{0} \in \mathbf{R}, \mathrm{c}_{\mathrm{V}}>0$
where

- $\mathrm{DI}(\mathrm{t})$ - the disposable income at the moment t ;
- $\mathrm{c}_{\mathrm{V}}$ - the marginal propensity to consume, $\mathrm{c}_{\mathrm{V}}=\frac{\mathrm{dC}}{\mathrm{dDI}}>0$;
- $\mathrm{C}_{0}$ is the intrinsic achieved autonomous consumption of households
(3) $\mathrm{G}(\mathrm{t})=\mathrm{i}_{\mathrm{G}} \mathrm{TI}(\mathrm{t}), \mathrm{i}_{\mathrm{G}} \in(0,1)$
where
- $\mathrm{TI}(\mathrm{t})$ - the total income at the moment t ;
- $\mathrm{i}_{\mathrm{G}}$ - the marginal index of final consumption of the government according to total income
(4) $\mathrm{TI}(\mathrm{t})=\mathrm{TR}(\mathrm{t})+\mathrm{OR}(\mathrm{t})$
where:
- $\operatorname{TR}(\mathrm{t})$ - tax rate at the moment t ;
- $\operatorname{OR}(\mathrm{t})$ - other revenues at the moment t
(5) $\mathrm{OR}(\mathrm{t})=\mathrm{i}_{\mathrm{OR}} \mathrm{Y}(\mathrm{t})+\mathrm{OR}_{0}, \mathrm{i}_{\mathrm{OR}} \in(0,1), \mathrm{OR}_{0} \in \mathbf{R}$
where:
- $\mathrm{Y}(\mathrm{t})$ - the output at the moment t ;
- $\mathrm{i}_{\mathrm{OR}}$ - the marginal index of other revenues according to the output;
- $\mathrm{OR}_{0}$ - the autonomous other revenues
(6) $\mathrm{I}(\mathrm{t})=\mathrm{i}_{\mathrm{Y}} \mathrm{Y}(\mathrm{t})+\mathrm{i}_{\mathrm{r}} \mathrm{r}(\mathrm{t}), \mathrm{i}_{\mathrm{Y}} \in(0,1), \mathrm{i}_{\mathrm{r}}<0$
where:
- $I(t)$ - investments at the moment $t$;
- $r(t)$ - the real interest rate at the moment $t$;
- $\mathrm{i}_{\mathrm{Y}}$ - the rate of investments;
- $\mathrm{i}_{\mathrm{r}}$ - a factor of influence on the investment rate
(7) $\operatorname{IM}(\mathrm{t})=\mathrm{im}_{\mathrm{Y}} \mathrm{Y}(\mathrm{t})+\mathrm{c}_{\mathrm{ei}} \mathrm{CH}(\mathrm{t})+\mathrm{IM}_{0}, \mathrm{im}_{\mathrm{Y}}>0, \mathrm{c}_{\mathrm{ei}}<0, \mathrm{IM}_{0} \in \mathbf{R}$
where:
- $\mathrm{CH}(\mathrm{t})$ - the exchange rate of the national currency based on the euro at the moment t;
- $\mathrm{im}_{\mathrm{Y}}$ - the rate of imports;
- $\mathrm{c}_{\mathrm{ei}}$ - a factor of imports influence on the exchange rate
- $\mathrm{IM}_{0}$ - the autonomous imports
(8) $E X(t)=\mathrm{ex}_{\mathrm{Y}} \mathrm{Y}(\mathrm{t})+\mathrm{c}_{\mathrm{ee}} \mathrm{CH}(\mathrm{t})+\mathrm{EX}_{0}, \mathrm{ex}_{\mathrm{Y}}>0, \mathrm{c}_{\mathrm{ee}}>0, \mathrm{EX}_{0} \in \mathbf{R}$
where:
- $\mathrm{ex}_{\mathrm{Y}}$ - the rate of exports;
- $\mathrm{C}_{\mathrm{ee}}$ - a factor of exports influence on the exchange rate
- $\mathrm{EX}_{0}$ - the autonomous exports
(9) $\mathrm{CH}(\mathrm{t})=\mathrm{r}_{\mathrm{CH}} \mathrm{t}+\mathrm{CH}_{0}, \mathrm{r}_{\mathrm{CH}}, \mathrm{CH}_{0} \in \mathbf{R}$
where:
- $\mathrm{r}_{\mathrm{CH}}$ - the marginal index of the exchange rate according to time;
- $\mathrm{CH}_{0}$ - the intercept of the regression
(10) $\mathrm{TF}(\mathrm{t})=\mathrm{c}_{\mathrm{TF}} \mathrm{Y}(\mathrm{t})+\mathrm{TF}_{0}, \mathrm{c}_{\mathrm{TF}} \in(0,1), \mathrm{TF}_{0} \in \mathbf{R}$
where:
- $\mathrm{TF}(\mathrm{t})$ - the government transfers at the moment t ;
- $\mathrm{c}_{\mathrm{TF}}$ - the marginal index of government transfers according to the output;
- $\mathrm{TF}_{0}$ - the autonomous government transfers
(11) $\mathrm{TR}(\mathrm{t})=\mathrm{t}_{\mathrm{Y}} \mathrm{Y}(\mathrm{t})+\mathrm{TR}_{0}, \mathrm{t}_{\mathrm{Y}} \in(0,1), \mathrm{TR}_{0} \in \mathbf{R}$
where:
- $\mathrm{t}_{\mathrm{Y}}$ - the marginal index of tax rate according to the output;
- $\mathrm{TR}_{0}$ - the intercept of the regression
(12) $\mathrm{DI}(\mathrm{t})=\mathrm{Y}(\mathrm{t})+\mathrm{TF}(\mathrm{t})-\mathrm{TR}(\mathrm{t})$
(13) $\mathrm{D}(\mathrm{t})=\mathrm{Y}(\mathrm{t})$ - the equation of equilibrium at the moment t
(14) $\mathrm{MD}(\mathrm{t})=\mathrm{md}_{\mathrm{Y}} \mathrm{Y}(\mathrm{t})+\mathrm{md}_{\mathrm{r}} \mathrm{r}(\mathrm{t}), \mathrm{md}_{\mathrm{Y}} \in(0,1), \mathrm{md}_{\mathrm{r}}<0$
where:
- $\mathrm{MD}(\mathrm{t})$ - the money demand in the economy at the moment t ;
- $\mathrm{md}_{\mathrm{Y}}$ - the rate of money demand in the economy;
- $\mathrm{md}_{\mathrm{r}}$ - a factor of influencing the demand for currency from the interest rate
(15) $\mathrm{MS}(\mathrm{t})=\mathrm{m}_{\mathrm{S}} \mathrm{t}+\mathrm{M}_{0}, \mathrm{~m}_{\mathrm{M}}, \mathrm{M}_{0} \in \mathbf{R}$
where:
- MS(t) - the money supply in the economy at the moment $t$;
- $\mathrm{m}_{\mathrm{S}}$ - the marginal index of the money supply according to time;
- $\mathrm{M}_{0}$ - the intercept of the regression
$M D(t)=M S(t)-$ the equation of equilibrium at the moment $t$.


## 3 The equilibrium at a fixed moment

From (4), (5), (11) we get:
(1) $\mathrm{TI}(\mathrm{t})=\left(\mathrm{t}_{\mathrm{Y}}+\mathrm{i}_{\mathrm{OR}}\right) \mathrm{Y}(\mathrm{t})+\mathrm{TR}_{0}+\mathrm{OR}_{0}$

From (3), (17):
(2) $\mathrm{G}(\mathrm{t})=\left(\mathrm{i}_{\mathrm{G}} \mathrm{t}_{\mathrm{Y}}+\mathrm{i}_{\mathrm{G}} \mathrm{i}_{\mathrm{OR}}\right) \mathrm{Y}(\mathrm{t})+\mathrm{i}_{\mathrm{G}}\left(\mathrm{TR}_{0}+\mathrm{OR}_{0}\right)$

From (7), (9):
(3) $\mathrm{IM}(\mathrm{t})=\mathrm{im}_{\mathrm{Y}} \mathrm{Y}(\mathrm{t})+\mathrm{C}_{\mathrm{ei}} \mathrm{I}_{\mathrm{CH}} \mathrm{t}+\mathrm{C}_{\mathrm{ei}} \mathrm{CH}_{0}+\mathrm{IM}_{0}$

From (8), (9):
(4) $\quad \mathrm{EX}(\mathrm{t})=\mathrm{ex}_{\mathrm{Y}} \mathrm{Y}(\mathrm{t})+\mathrm{C}_{\mathrm{ee}} \mathrm{r}_{\mathrm{CH}} \mathrm{t}+\mathrm{C}_{\mathrm{ee}} \mathrm{CH}_{0}+\mathrm{EX}_{0}$

From (10), (11), (12) we get:
(5) $\mathrm{DI}(\mathrm{t})=\left(1+\mathrm{c}_{\mathrm{TF}}-\mathrm{t}_{\mathrm{Y}}\right) \mathrm{Y}(\mathrm{t})+\mathrm{TF}_{0}-\mathrm{TR}_{0}$

From (2), (21):
(6) $\mathrm{C}(\mathrm{t})=\left(\mathrm{C}_{\mathrm{V}}+\mathrm{C}_{\mathrm{V}} \mathrm{C}_{\mathrm{TF}}-\mathrm{C}_{\mathrm{V}} \mathrm{t}_{\mathrm{Y}}\right) \mathrm{Y}(\mathrm{t})+\mathrm{C}_{\mathrm{V}}\left(\mathrm{TF}_{0}-\mathrm{TR}_{0}\right)+\mathrm{C}_{0}$ Now, from (1), (6), (18), (19), (20), (22) we have:
(7) $\mathrm{D}(\mathrm{t})=\left(\mathrm{C}_{\mathrm{V}}+\mathrm{C}_{\mathrm{V}} \mathrm{C}_{\mathrm{TF}}-\mathrm{C}_{\mathrm{V}} \mathrm{t}_{\mathrm{Y}}+\mathrm{i}_{\mathrm{G}} \mathrm{t}_{\mathrm{Y}}+\mathrm{i}_{\mathrm{G}} \mathrm{i}_{\mathrm{OR}}+\mathrm{i}_{\mathrm{Y}}+\mathrm{eX}_{\mathrm{Y}}-\mathrm{im}_{\mathrm{Y}}\right) \mathrm{Y}(\mathrm{t})+\mathrm{i}_{\mathrm{r}} \mathrm{r}(\mathrm{t})+\left(\mathrm{c}_{\mathrm{ee}} \mathrm{r}_{\mathrm{CH}}-\mathrm{C}_{\mathrm{e}} \mathrm{r}_{\mathrm{CH}}\right) \mathrm{t}+$ $\mathrm{C}_{\mathrm{V}}\left(\mathrm{TF}_{0}-\mathrm{TR}_{0}\right)+\mathrm{i}_{\mathrm{G}}\left(\mathrm{TR}_{0}+\mathrm{OR}_{0}\right)+\left(\mathrm{C}_{\mathrm{ee}}-\mathrm{C}_{\mathrm{e}}\right) \mathrm{CH}_{0}+\mathrm{C}_{0}+\mathrm{EX}_{0}-\mathrm{IM}_{0}$

From (13) and (23) we get the first equation of the equilibrium:
(8) $\left(\mathrm{c}_{\mathrm{Y}}+\mathrm{C}_{\mathrm{V}} \mathrm{C}_{\mathrm{TF}}-\mathrm{C}_{\mathrm{V}} \mathrm{t}_{\mathrm{Y}}+\mathrm{i}_{\mathrm{G}} \mathrm{t}_{\mathrm{Y}}+\mathrm{i}_{\mathrm{G}} \mathrm{i}_{\mathrm{OR}}+\mathrm{i}_{\mathrm{Y}}+\mathrm{ex}_{\mathrm{Y}}-\mathrm{im}_{\mathrm{Y}}-1\right) \mathrm{Y}(\mathrm{t})+\mathrm{i}_{\mathrm{r}}\left(\mathrm{t}(\mathrm{t})+\left(\mathrm{cee}_{\mathrm{ee}} \mathrm{r}_{\mathrm{CH}}-\mathrm{Ceil}_{\mathrm{e}} \mathrm{I}_{\mathrm{H}}\right) \mathrm{t}+\right.$
$\mathrm{c}_{\mathrm{V}}\left(\mathrm{TF}_{0}-\mathrm{TR}_{0}\right)+\mathrm{i}_{\mathrm{G}}\left(\mathrm{TR}_{0}+\mathrm{OR}_{0}\right)+\left(\mathrm{C}_{\mathrm{ee}}-\mathrm{C}_{\mathrm{e}}\right) \mathrm{CH}_{0}+\mathrm{C}_{0}+\mathrm{EX}_{0}-\mathrm{IM}_{0}=0$
and from (14), (15), (16) we get the second equation of the equilibrium
(9) $\mathrm{md}_{\mathrm{Y}} \mathrm{Y}(\mathrm{t})+\mathrm{md}_{\mathrm{r}}(\mathrm{t})-\mathrm{m}_{\mathrm{s}}-\mathrm{M}_{0}=0$

Let note now:
(10) $\alpha=\mathrm{c}_{\mathrm{V}}+\mathrm{c}_{\mathrm{V}} \mathrm{c}_{\mathrm{TF}}-\mathrm{C}_{\mathrm{V}} \mathrm{t}_{\mathrm{Y}}+\mathrm{i}_{\mathrm{G}} \mathrm{t}_{\mathrm{Y}}+\mathrm{i}_{\mathrm{G}} \mathrm{i}_{\mathrm{OR}}+\mathrm{i}_{\mathrm{Y}}+\mathrm{e}_{\mathrm{Y}}-\mathrm{im}_{\mathrm{Y}}-1$
$\beta=\mathrm{C}_{\mathrm{ei}} \mathrm{I}_{\mathrm{CH}}-\mathrm{Cee}_{\mathrm{Ce}} \mathrm{T}_{\mathrm{CH}}$
$\gamma=\mathrm{c}_{\mathrm{V}}\left(\mathrm{TF}_{0}-\mathrm{TR}_{0}\right)+\mathrm{i}_{\mathrm{G}}\left(\mathrm{TR}_{0}+\mathrm{OR}_{0}\right)+\left(\mathrm{C}_{\mathrm{ee}}-\mathrm{C}_{\mathrm{e}}\right) \mathrm{CH}_{0}+\mathrm{C}_{0}+\mathrm{EX}_{0}-\mathrm{IM}_{0}$
The equilibrium equations become:
(11) $\left\{\begin{array}{l}\alpha \mathrm{Y}(\mathrm{t})+\mathrm{i}_{\mathrm{r}} \mathrm{r}(\mathrm{t})=\beta \mathrm{t}-\gamma \\ \mathrm{md}_{\mathrm{Y}} \mathrm{Y}(\mathrm{t})+\mathrm{md}_{\mathrm{r}} \mathrm{r}(\mathrm{t})=\mathrm{m}_{\mathrm{s}} \mathrm{t}+\mathrm{M}_{0}\end{array}\right.$

The solutions of equilibrium are (noted with same symbols without being a confusion):
(12) $\left\{\begin{array}{l}Y(t)=\frac{\left(\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}\right)}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}} \mathrm{t}-\frac{\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right)}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}} \\ \mathrm{r}(\mathrm{t})=\frac{\left(\mathrm{m}_{\mathrm{S}} \alpha-\beta \mathrm{md}_{\mathrm{Y}}\right)}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathbf{i}_{\mathrm{r}}} \mathrm{t}+\frac{\left(\gamma \mathrm{md}_{\mathrm{Y}}+\alpha \mathrm{M}_{0}\right)}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}\end{array}\right.$

At equilibrium, replacing (28) in (1)-(16), we have:

$$
\begin{aligned}
& \mathrm{C}(\mathrm{t})=\frac{\left(\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}\right)+\left(\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}_{\mathrm{r}}}\right)_{\mathrm{C}_{\mathrm{TF}}}-\left(\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}\right) \mathrm{t}_{\mathrm{Y}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}} \mathrm{C}_{\mathrm{V}} \mathrm{t}+ \\
& \text { (13) } \frac{\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right) \mathrm{t}_{\mathrm{Y}}-\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right)-\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right) \mathrm{c}_{\mathrm{TF}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}+\mathrm{c}_{\mathrm{V}}\left(\mathrm{TF}_{0}-\mathrm{TR}_{0}\right)+\mathrm{C}_{0}
\end{aligned}
$$

(14) $\mathrm{G}(\mathrm{t})=\mathrm{i}_{\mathrm{G}} \mathrm{TI}(\mathrm{t})=$
$\frac{\left(\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}_{\mathrm{r}}}\right) \mathrm{t}_{\mathrm{Y}}+\left(\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}\right) \mathrm{i}_{\mathrm{OR}} \mathrm{i}_{\mathrm{G}} \mathrm{t}-\frac{\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right) \mathrm{t}_{\mathrm{Y}}+\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right) \mathrm{i}_{\mathrm{OR}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}} \mathrm{i}_{\mathrm{G}}+\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}{\alpha}$
$\left(\mathrm{TR}_{0}+\mathrm{OR}_{0}\right) \mathrm{i}_{\mathrm{G}}$
(15) $\mathrm{TI}(\mathrm{t})=$

$$
\begin{aligned}
& \frac{\left(\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}\right) \mathrm{t}_{\mathrm{Y}}+\left(\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}\right) \mathrm{i}_{\mathrm{OR}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}} \mathrm{t}-\frac{\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right) \mathrm{t}_{\mathrm{Y}}+\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right) \mathrm{i}_{\mathrm{OR}}}{\alpha \mathrm{md}_{\mathrm{r}}-\operatorname{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}+ \\
& \mathrm{TR}_{0}+\mathrm{OR}_{0}
\end{aligned}
$$

(16) $\mathrm{OR}(\mathrm{t})=\mathrm{i}_{\mathrm{OR}} \mathrm{Y}(\mathrm{t})+\mathrm{OR}_{0}=\frac{\left(\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}\right) \mathrm{i}_{\mathrm{OR}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}} \mathrm{t}-\frac{\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right) \mathrm{i}_{\mathrm{OR}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}+O \mathrm{OR}_{0}$
(17) $\mathrm{I}(\mathrm{t})=$

$$
\frac{\left(\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}\right) \mathrm{i}_{\mathrm{Y}}+\left(\mathrm{m}_{\mathrm{S}} \alpha-\beta \mathrm{md}_{\mathrm{Y}}\right) \mathrm{i}_{\mathrm{r}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}} \mathrm{t}+\frac{\left(\gamma \mathrm{md}_{\mathrm{Y}}+\alpha \mathrm{M}_{0}\right) \mathrm{i}_{\mathrm{r}}-\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md} \mathrm{r}_{\mathrm{r}}\right) \mathrm{i}_{\mathrm{Y}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}
$$

(18) $\mathrm{IM}(\mathrm{t})=$

$$
\frac{\left(\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}\right) \mathrm{im}_{\mathrm{Y}}+\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right) \mathrm{C}_{\mathrm{ei}} \mathrm{r}_{\mathrm{CH}}}{\alpha m d_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}} \mathrm{t}-\frac{\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right) \mathrm{im}_{\mathrm{Y}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}+\mathrm{c}_{\mathrm{ei}} \mathrm{CH}_{0}+\mathrm{IM}_{0}
$$

(19) $\mathrm{EX}(\mathrm{t})=$

$$
\frac{\left(\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}\right) \mathrm{ex}_{\mathrm{Y}}+\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right) \mathrm{c}_{\mathrm{ee}} \mathrm{r}_{\mathrm{CH}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}} \mathrm{t}-\frac{\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right) \mathrm{ex}_{\mathrm{Y}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}+\mathrm{c}_{\mathrm{ee}} \mathrm{CH}_{0}+\mathrm{EX}_{0}
$$

(20) $\mathrm{CH}(\mathrm{t})=\mathrm{r}_{\mathrm{CH}} \mathrm{t}+\mathrm{CH}_{0}$
(21) $\mathrm{TF}(\mathrm{t})=\frac{\left(\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}\right) \mathrm{c}_{\mathrm{TF}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}} \mathrm{t}-\frac{\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right) \mathrm{c}_{\mathrm{TF}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}+\mathrm{TF}_{0}$
(22) $\operatorname{TR}(\mathrm{t})=\frac{\left(\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}\right) \mathrm{t}_{\mathrm{Y}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}} \mathrm{t}-\frac{\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right) \mathrm{t}_{\mathrm{Y}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}+\mathrm{TR}_{0}$

$$
\mathrm{DI}(\mathrm{t})=\frac{\left(\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}\right)+\left(\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}\right) \mathrm{c}_{\mathrm{TF}}-\left(\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}\right) \mathrm{t}_{\mathrm{Y}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}} \mathrm{t}+
$$

(23)

$$
\frac{\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right) \mathrm{t}_{\mathrm{Y}}-\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right)-\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right) \mathrm{c}_{\mathrm{TF}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}+\mathrm{TF}_{0}-\mathrm{TR}_{0}
$$

$\mathrm{MD}(\mathrm{t})=\mathrm{MS}(\mathrm{t})=\mathrm{m}_{\mathrm{s}} \mathrm{t}+\mathrm{M}_{0}$.

## 4 The variations of equilibrium output and real interest rate based on the parameter values

First of all, we will compute the derivatives of functions $\alpha, \beta$ and $\gamma$ in function of the parameters of the model.

$$
\begin{aligned}
& \text { (16) } \frac{\partial \alpha}{\partial \mathrm{c}_{\mathrm{V}}}=1+\mathrm{c}_{\mathrm{TF}}-\mathrm{t}_{\mathrm{Y}}, \frac{\partial \alpha}{\partial \mathrm{C}_{0}}=0, \frac{\partial \alpha}{\partial \mathrm{i}_{\mathrm{G}}}=\mathrm{t}_{\mathrm{Y}}+\mathrm{i}_{\mathrm{OR}}, \frac{\partial \alpha}{\partial \mathrm{i}_{\mathrm{OR}}}=\mathrm{i}_{\mathrm{G}}, \frac{\partial \alpha}{\partial \mathrm{OR}_{0}}=0, \frac{\partial \alpha}{\partial \mathrm{i}_{\mathrm{Y}}}=1, \\
& \frac{\partial \alpha}{\partial \mathrm{i}_{\mathrm{r}}}=0, \\
& \frac{\partial \alpha}{\partial \mathrm{im}_{\mathrm{Y}}}=-1, \quad \frac{\partial \alpha}{\partial \mathrm{c}_{\mathrm{ei}}}=0, \quad \frac{\partial \alpha}{\partial \mathrm{IM}_{0}}=0, \quad \frac{\partial \alpha}{\partial \mathrm{ex}_{\mathrm{Y}}}=1, \quad \frac{\partial \alpha}{\partial \mathrm{c}_{\mathrm{ee}}}=0, \quad \frac{\partial \alpha}{\partial \mathrm{EX}_{0}}=0, \quad \frac{\partial \alpha}{\partial \mathrm{r}_{\mathrm{CH}}}=0, \\
& \frac{\partial \alpha}{\partial \mathrm{CH}_{0}}=0, \quad \frac{\partial \alpha}{\partial \mathrm{c}_{\mathrm{TF}}}=\mathrm{c}_{\mathrm{V}}, \quad \frac{\partial \alpha}{\partial \mathrm{TF}_{0}}=0, \quad \frac{\partial \alpha}{\partial \mathrm{t}_{\mathrm{Y}}}=\mathrm{i}_{\mathrm{G}}-\mathrm{c}_{\mathrm{V}}, \quad \frac{\partial \alpha}{\partial \mathrm{TR}_{0}}=0, \quad \frac{\partial \alpha}{\partial \mathrm{md}_{\mathrm{Y}}}=0, \\
& \frac{\partial \alpha}{\partial \mathrm{md}_{\mathrm{r}}}=0, \frac{\partial \alpha}{\partial \mathrm{~m}_{\mathrm{S}}}=0, \frac{\partial \alpha}{\partial \mathrm{M}_{0}}=0
\end{aligned}
$$

(17) $\frac{\partial \beta}{\partial \mathrm{c}_{\mathrm{V}}}=0, \frac{\partial \beta}{\partial \mathrm{C}_{0}}=0, \frac{\partial \beta}{\partial \mathrm{i}_{\mathrm{G}}}=0, \frac{\partial \beta}{\partial \mathrm{i}_{\mathrm{OR}}}=0, \frac{\partial \beta}{\partial \mathrm{OR}_{0}}=0, \frac{\partial \beta}{\partial \mathrm{i}_{\mathrm{Y}}}=0, \frac{\partial \beta}{\partial \mathrm{i}_{\mathrm{r}}}=0, \frac{\partial \beta}{\partial \mathrm{im}_{\mathrm{Y}}}=0$,

$$
\begin{aligned}
& \frac{\partial \beta}{\partial \mathrm{c}_{\mathrm{ei}}}=\mathrm{r}_{\mathrm{CH}}, \frac{\partial \beta}{\partial \mathrm{I}_{0}}=0, \frac{\partial \beta}{\partial \mathrm{ex}_{\mathrm{Y}}}=0, \frac{\partial \beta}{\partial \mathrm{c}_{\mathrm{ee}}}=-\mathrm{r}_{\mathrm{CH}}, \frac{\partial \beta}{\partial \mathrm{E}}=0, \frac{\partial \beta}{\partial \mathrm{r}_{\mathrm{CH}}}=\mathrm{c}_{\mathrm{ei}}-\mathrm{c}_{\mathrm{ee}}, \\
& \frac{\partial \beta}{\partial \mathrm{CH}_{0}}=0, \frac{\partial \beta}{\partial \mathrm{c}_{\mathrm{TF}}}=0, \frac{\partial \beta}{\partial \mathrm{TF}_{0}}=0, \frac{\partial \beta}{\partial \mathrm{t}_{\mathrm{Y}}}=0, \frac{\partial \beta}{\partial \mathrm{TR}_{0}}=0, \frac{\partial \beta}{\partial \mathrm{md}_{\mathrm{Y}}}=0, \frac{\partial \beta}{\partial \mathrm{md}_{\mathrm{r}}}=0, \\
& \frac{\partial \beta}{\partial \mathrm{~m}_{\mathrm{S}}}=0, \frac{\partial \beta}{\partial \mathrm{M}_{0}}=0
\end{aligned}
$$

(18) $\frac{\partial \gamma}{\partial \mathrm{c}_{\mathrm{V}}}=\mathrm{TF}_{0}-\mathrm{TR}_{0}, \quad \frac{\partial \gamma}{\partial \mathrm{C}_{0}}=1, \quad \frac{\partial \gamma}{\partial \mathrm{i}_{\mathrm{G}}}=\mathrm{TR}_{0}+\mathrm{OR}_{0}, \quad \frac{\partial \gamma}{\partial \mathrm{i}_{\mathrm{OR}}}=0, \quad \frac{\partial \gamma}{\partial \mathrm{OR}_{0}}=\mathrm{i}_{\mathrm{G}}$,

$$
\begin{aligned}
& \frac{\partial \gamma}{\partial \mathrm{i}_{\mathrm{Y}}}=0, \quad \frac{\partial \gamma}{\partial \mathrm{i}_{\mathrm{r}}}=0, \quad \frac{\partial \gamma}{\partial \mathrm{im}_{\mathrm{Y}}}=0, \quad \frac{\partial \gamma}{\partial \mathrm{c}_{\mathrm{ei}}}=-\mathrm{CH}_{0}, \quad \frac{\partial \gamma}{\partial \mathrm{IM}_{0}}=-1, \quad \frac{\partial \gamma}{\partial \mathrm{ex}_{\mathrm{Y}}}=0, \\
& \frac{\partial \gamma}{\partial \mathrm{c}_{\mathrm{ee}}}=\mathrm{CH}_{0}, \frac{\partial \gamma}{\partial \mathrm{EX}_{0}}=1, \frac{\partial \gamma}{\partial \mathrm{r}_{\mathrm{CH}}}=0, \frac{\partial \gamma}{\partial \mathrm{CH}_{0}}=\mathrm{c}_{\mathrm{ee}}-\mathrm{c}_{\mathrm{ei}}, \frac{\partial \gamma}{\partial \mathrm{c}_{\mathrm{TF}}}=0, \quad \frac{\partial \gamma}{\partial \mathrm{TF}_{0}}=\mathrm{c}_{\mathrm{V}}, \\
& \frac{\partial \gamma}{\partial \mathrm{t}_{\mathrm{Y}}}=0, \\
& \frac{\partial \gamma}{\partial \mathrm{TR}_{0}}=-\mathrm{c}_{\mathrm{V}}, \frac{\partial \gamma}{\partial \mathrm{md}_{\mathrm{Y}}}=0, \frac{\partial \gamma}{\partial \mathrm{md}_{\mathrm{r}}}=0, \frac{\partial \gamma}{\partial \mathrm{~m}_{\mathrm{S}}}=0, \frac{\partial \gamma}{\partial \mathrm{M}_{0}}=0
\end{aligned}
$$

We have now:
(19)

$$
\frac{\partial \mathrm{Y}}{\partial \mathrm{c}_{\mathrm{V}}}=-\frac{\left(\mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}\right)\left(1+\mathrm{c}_{\mathrm{TF}}-\mathrm{t}_{\mathrm{Y}}\right)}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{md}_{\mathrm{r}} \mathrm{t}+
$$

$$
\operatorname{md}_{\mathrm{r}}\left(\frac{\mathrm{TF}_{0}-\mathrm{TR}_{0}}{\alpha_{\mathrm{md}}^{\mathrm{r}}} \mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}} \quad-\frac{\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right)\left(1+\mathrm{c}_{\mathrm{TF}}-\mathrm{t}_{\mathrm{Y}}\right)}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}}\right)
$$

(20) $\frac{\partial \mathrm{Y}}{\partial \mathrm{C}_{0}}=-\frac{\mathrm{md}_{\mathrm{r}}}{\alpha \mathrm{md}_{\mathrm{r}}-\text { md }_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}$
$\frac{\partial \mathrm{Y}}{\partial \mathrm{i}_{\mathrm{G}}}=-\frac{\left(\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}\right)\left(\mathrm{t}_{\mathrm{Y}}+\mathrm{i}_{\mathrm{OR}}\right)}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{md}_{\mathrm{r}} \mathrm{t}+$
(21)
$\operatorname{md}_{\mathrm{r}}\left(\frac{\mathrm{TR}_{0}+\mathrm{OR}_{0}}{\alpha \mathrm{md}_{\mathrm{r}}-\text { md }_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}-\frac{\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right)\left(\mathrm{t}_{\mathrm{Y}}+\mathrm{i}_{\mathrm{OR}}\right)}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}}\right)$
(22) $\frac{\partial Y}{\partial i_{O R}}=-\frac{\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{i}_{\mathrm{G}} \mathrm{md}_{\mathrm{r}} \mathrm{t}+\frac{\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{i}_{\mathrm{G}} \mathrm{md}_{\mathrm{r}}$
(23) $\frac{\partial \mathrm{Y}}{\partial \mathrm{OR}_{0}}=-\frac{\mathrm{i}_{\mathrm{G}} \mathrm{md}_{\mathrm{r}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}$
(24) $\frac{\partial \mathrm{Y}}{\partial \mathrm{i}_{\mathrm{Y}}}=-\frac{\left(\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}\right)}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{md}_{\mathrm{r}} \mathrm{t}+\frac{\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right)}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{md}_{\mathrm{r}}$
(25) $\frac{\partial \mathrm{Y}}{\partial \mathrm{i}_{\mathrm{r}}}=\frac{\beta \mathrm{md}_{\mathrm{Y}}-\alpha \mathrm{m}_{\mathrm{S}}}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{md}_{\mathrm{r}} \mathrm{t}-\frac{\gamma \mathrm{md}_{\mathrm{Y}}+\alpha \mathrm{M}_{0}}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{md}_{\mathrm{r}}$
(26) $\frac{\partial \mathrm{Y}}{\partial \mathrm{m}_{\mathrm{Y}}}=\frac{\left(\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{i}_{\mathrm{r}}}\right)}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{md}_{\mathrm{r}} \mathrm{t}-\frac{\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right)}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{md}_{\mathrm{r}}$
(27) $\frac{\partial Y}{\partial c_{e i}}=\frac{r_{C H} m d_{r}}{\alpha m d_{r}-\text { md }_{Y} i_{r}} t+\frac{\operatorname{md}_{r} C H_{0}}{\alpha \operatorname{md}_{r}-\operatorname{md}_{Y} i_{r}}$
(28) $\frac{\partial \mathrm{Y}}{\partial \mathrm{IM}_{0}}=-\frac{\mathrm{md}_{\mathrm{r}}}{\alpha \mathrm{md}_{\mathrm{r}}-\operatorname{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}$
(29) $\frac{\partial \mathrm{Y}}{\partial \mathrm{ex}_{\mathrm{Y}}}=-\frac{\left(\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}_{\mathrm{r}}}\right)}{\left(\alpha \mathrm{i}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{md}_{\mathrm{r}} \mathrm{t}+\frac{\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right)}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{md}_{\mathrm{r}}$
(30) $\frac{\partial \mathrm{Y}}{\partial \mathrm{c}_{\mathrm{ee}}}=-\frac{\mathrm{r}_{\mathrm{CH}} \mathrm{md}_{\mathrm{r}}}{\alpha \mathrm{md}_{\mathrm{r}}-\text { md }_{Y} \mathrm{i}_{\mathrm{r}}} \mathrm{t}-\frac{\mathrm{md}_{\mathrm{r}} \mathrm{CH}_{0}}{\alpha \mathrm{md}_{\mathrm{r}}-\operatorname{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}$
(31) $\frac{\partial \mathrm{Y}}{\partial \mathrm{EX}_{0}}=-\frac{\mathrm{md}_{\mathrm{r}}}{\alpha \mathrm{md}_{\mathrm{r}}-\operatorname{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}$
(32) $\frac{\partial \mathrm{Y}}{\partial \mathrm{r}_{\mathrm{CH}}}=\frac{\mathrm{md}_{\mathrm{r}}\left(\mathrm{c}_{\mathrm{ei}}-\mathrm{c}_{\mathrm{ee}}\right)}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}} \mathrm{t}$
(33) $\frac{\partial \mathrm{Y}}{\partial \mathrm{CH}_{0}}=-\frac{\mathrm{md}_{\mathrm{r}}\left(\mathrm{c}_{\mathrm{ee}}-\mathrm{c}_{\mathrm{ei}}\right)}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}$
(34) $\frac{\partial \mathrm{Y}}{\partial \mathrm{c}_{\mathrm{TF}}}=-\frac{\left(\mathrm{\beta md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}_{\mathrm{r}}} \mathrm{i}_{\mathrm{r}}\right)}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{c}_{\mathrm{V}} \mathrm{md}_{\mathrm{r}} \mathrm{t}+\frac{\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right)}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{c}_{\mathrm{V}} \mathrm{md}_{\mathrm{r}}$
(35) $\frac{\partial \mathrm{Y}}{\partial \mathrm{TF}_{0}}=-\frac{\mathrm{c}_{\mathrm{V}} \mathrm{md}_{\mathrm{r}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}$
(36) $\frac{\partial \mathrm{Y}}{\partial \mathrm{t}_{\mathrm{Y}}}=-\frac{\left(\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}\right)\left(\mathrm{i}_{\mathrm{G}}-\mathrm{c}_{\mathrm{V}}\right)}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{md}_{\mathrm{r}} \mathrm{t}+\frac{\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right)\left(\mathrm{i}_{\mathrm{G}}-\mathrm{c}_{\mathrm{V}}\right)}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{md}_{\mathrm{r}}$
(37) $\frac{\partial \mathrm{Y}}{\partial \mathrm{TR}_{0}}=\frac{\mathrm{C}_{\mathrm{V}} \mathrm{md}_{\mathrm{r}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}$
(38) $\frac{\partial Y}{\partial m d_{Y}}=\frac{\left(\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}\right)}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{i}_{\mathrm{r}} \mathrm{t}-\frac{\left(\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}\right)}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{i}_{\mathrm{r}}$
(39) $\frac{\partial \mathrm{Y}}{\partial \mathrm{md}_{\mathrm{r}}}=\frac{\mathrm{m}_{\mathrm{S}} \alpha-\beta \mathrm{md}_{\mathrm{Y}}}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{i}_{\mathrm{r}} \mathrm{t}+\frac{\gamma \mathrm{md}_{\mathrm{Y}}+\alpha \mathrm{M}_{0}}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{i}_{\mathrm{r}}$
(40) $\frac{\partial \mathrm{Y}}{\partial \mathrm{m}_{\mathrm{S}}}=-\frac{\mathrm{i}_{\mathrm{r}}}{\alpha \mathrm{md}_{\mathrm{r}}-\operatorname{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}} \mathrm{t}$
(41) $\frac{\partial \mathrm{Y}}{\partial \mathrm{M}_{0}}=-\frac{\mathrm{i}_{\mathrm{r}}}{\alpha \mathrm{md}_{\mathrm{r}}-\operatorname{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}$
(42)

$$
\begin{aligned}
& \frac{\partial r}{\partial c_{V}}=\frac{\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{md}_{\mathrm{Y}}\left(1+\mathrm{c}_{\mathrm{TF}}-\mathrm{t}_{\mathrm{Y}}\right) \mathrm{t}+\frac{\mathrm{TF}_{0}-\mathrm{TR}_{0}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}} \mathrm{md}_{\mathrm{Y}}- \\
& \frac{\left(\mathrm{i}_{\mathrm{r}} \mathrm{M}_{0}+\gamma \mathrm{md}_{\mathrm{r}}\right)\left(1+\mathrm{c}_{\mathrm{TF}}-\mathrm{t}_{\mathrm{Y}}\right)}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{md}_{\mathrm{Y}}
\end{aligned}
$$

(43) $\frac{\partial \mathrm{r}}{\partial \mathrm{C}_{0}}=\frac{\mathrm{md}_{\mathrm{Y}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}$
(44)

$$
\frac{\partial r}{\partial i_{G}}=\frac{\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{md}_{\mathrm{Y}}\left(\mathrm{t}_{\mathrm{Y}}+\mathrm{i}_{\mathrm{OR}}\right) \mathrm{t}+\frac{\mathrm{md}_{\mathrm{Y}}\left(\mathrm{TR}_{0}+\mathrm{OR}_{0}\right)+\mathrm{M}_{0}\left(\mathrm{t}_{\mathrm{Y}}+\mathrm{i}_{\mathrm{OR}}\right)}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}
$$

$$
-\frac{\left(\gamma \mathrm{md}_{\mathrm{Y}}+\alpha \mathrm{M}_{0}\right)\left(\mathrm{t}_{\mathrm{Y}}+\mathrm{i}_{\mathrm{OR}}\right)}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{md}_{\mathrm{r}}
$$


(46) $\frac{\partial r}{\partial \mathrm{OR}_{0}}=\frac{\mathrm{i}_{\mathrm{G}} \mathrm{md}_{\mathrm{Y}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}$
(47) $\frac{\partial r}{\partial i_{Y}}=\frac{\beta m d_{r}-m_{S} i_{r}}{\left(\alpha m d_{r}-m d_{Y} i_{r}\right)^{2}} m d_{Y} t-\frac{M_{0} i_{r}+\gamma m d_{r}}{\left(\alpha m d_{r}-\operatorname{md}_{Y} i_{r}\right)^{2}}{m d_{Y}}^{(\alpha)^{2}}$
(48) $\frac{\partial \mathrm{r}}{\partial \mathrm{i}_{\mathrm{r}}}=\frac{\mathrm{m}_{\mathrm{S}} \alpha-\beta \mathrm{md}_{\mathrm{Y}}}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{md}_{\mathrm{Y}} \mathrm{t}+\frac{\gamma \mathrm{md}_{\mathrm{Y}}+\alpha \mathrm{M}_{0}}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{md}_{\mathrm{Y}}$
(49) $\frac{\partial r}{\partial \mathrm{im}_{\mathrm{Y}}}=\frac{\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}-\beta \mathrm{md}_{\mathrm{r}}}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{md}_{\mathrm{Y}} \mathrm{t}+\frac{\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{md}_{\mathrm{Y}}$
(50) $\frac{\partial r}{\partial c_{e i}}=-\frac{r_{C H} m d_{Y}}{\alpha m d_{r}-\text { md }_{Y} i_{r}} t-\frac{\mathrm{md}_{\mathrm{Y}} \mathrm{CH}_{0}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}$
(51) $\frac{\partial \mathrm{r}}{\partial \mathrm{IM}_{0}}=-\frac{\mathrm{md}_{\mathrm{Y}}}{\alpha \mathrm{md}_{\mathrm{r}}-\operatorname{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}$
(52) $\frac{\partial r}{\partial \mathrm{ex}_{\mathrm{Y}}}=\frac{\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{md}_{\mathrm{Y}} \mathrm{t}-\frac{\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{md}_{\mathrm{Y}}$
(53) $\frac{\partial \mathrm{r}}{\partial \mathrm{c}_{\mathrm{ee}}}=\frac{\mathrm{r}_{\mathrm{CH}} \mathrm{md}_{\mathrm{Y}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}} \mathrm{t}+\frac{\mathrm{md}_{\mathrm{Y}} \mathrm{CH}_{0}}{\alpha \operatorname{add}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}$
(54) $\frac{\partial r}{\partial E X_{0}}=\frac{m d_{Y}}{\alpha \operatorname{md}_{r}-\operatorname{md}_{Y} i_{r}}$
(55) $\frac{\partial \mathrm{r}}{\partial \mathrm{r}_{\mathrm{CH}}}=\frac{\mathrm{c}_{\mathrm{ee}}-\mathrm{C}_{\mathrm{ei}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}} \mathrm{md}_{\mathrm{Y}} \mathrm{t}$
(56) $\frac{\partial r}{\partial \mathrm{CH}_{0}}=\frac{\mathrm{c}_{\text {ee }}-\mathrm{C}_{\text {ei }}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}} \mathrm{md}_{\mathrm{Y}}$
(57) $\frac{\partial r}{\partial c_{T F}}=\frac{\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{c}_{\mathrm{V}} \mathrm{md}_{\mathrm{Y}} \mathrm{t}-\frac{\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \mathrm{md}_{\mathrm{Y}} \mathrm{c}_{\mathrm{V}}$
(58) $\frac{\partial \mathrm{r}}{\partial \mathrm{TF}_{0}}=\frac{\mathrm{c}_{\mathrm{V}} \mathrm{md}_{\mathrm{Y}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}$
(59) $\frac{\partial r}{\partial t_{Y}}=\frac{\beta \mathrm{md}_{\mathrm{r}}-\mathrm{m}_{\mathrm{S}} \mathrm{i}_{\mathrm{r}}}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}}\left(\mathrm{i}_{\mathrm{G}}-\mathrm{c}_{\mathrm{V}}\right) \mathrm{md} \mathrm{Y}_{\mathrm{Y}} \mathrm{t}-\frac{\mathrm{M}_{0} \mathrm{i}_{\mathrm{r}}+\gamma \mathrm{md}_{\mathrm{r}}}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}}\left(\mathrm{i}_{\mathrm{G}}-\mathrm{c}_{\mathrm{V}}\right) \mathrm{md} \mathrm{Y}_{\mathrm{Y}}$
(60) $\frac{\partial \mathrm{r}}{\partial \mathrm{TR}_{0}}=-\frac{\mathrm{c}_{\mathrm{V}} \mathrm{md}_{\mathrm{Y}}}{\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}$
(61) $\frac{\partial r}{\partial m d_{Y}}=\frac{m_{S} i_{r}-\beta m d_{r}}{\left(\alpha m d_{r}-m d_{Y} i_{r}\right)^{2}} \alpha t+\frac{\gamma m d_{r}+M_{0} i_{r}}{\left(\alpha m d_{r}-m d_{Y} i_{r}\right)^{2}} \alpha$
(62) $\frac{\partial r}{\partial \mathrm{md}_{\mathrm{r}}}=\frac{\beta \mathrm{md}_{\mathrm{Y}}-\mathrm{m}_{\mathrm{S}} \alpha}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \alpha \mathrm{t}-\frac{\gamma \mathrm{md}_{\mathrm{Y}}+\alpha \mathrm{M}_{0}}{\left(\alpha \mathrm{md}_{\mathrm{r}}-\mathrm{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}\right)^{2}} \alpha$
(63) $\frac{\partial r}{\partial m_{S}}=\frac{\alpha}{\alpha m d_{r}-m d_{Y} i_{r}} t$
(64) $\frac{\partial r}{\partial M_{0}}=\frac{\alpha}{\alpha \mathrm{md}_{\mathrm{r}}-\operatorname{md}_{\mathrm{Y}} \mathrm{i}_{\mathrm{r}}}$

## 5 The variations of equilibrium output and real interest rate based on the parameter values

In what follows let consider the dynamic equations of the model:
(65) $\left\{\begin{array}{l}\frac{d Y}{d t}=A(D(t)-Y(t)) \\ \frac{d r}{d t}=B(M D(t)-M S(t))\end{array}, A, B \in \mathbf{R}\right.$

With notations (26) we have:
(66) $\mathrm{D}(\mathrm{t})=(\alpha+1) \mathrm{Y}(\mathrm{t})+\mathrm{i}_{\mathrm{r}} \mathrm{r}(\mathrm{t})-\beta \mathrm{t}+\gamma$
(67) $\mathrm{MD}(\mathrm{t})=\mathrm{md}_{\mathrm{Y}} \mathrm{Y}(\mathrm{t})+\mathrm{md}_{\mathrm{r}} \mathrm{r}(\mathrm{t})$

The system becomes:
(68)

$$
\left\{\begin{array}{l}
\frac{d Y}{d t}=A \alpha Y(t)+A i_{r} r(t)-A \beta t+A \gamma \\
\frac{d r}{d t}=\operatorname{Bmd}_{Y} Y(t)+\operatorname{Bmd}_{r} r(t)-B m_{s} t-B M_{0}
\end{array}\right.
$$

or, in matrix notation:

and it is a system of differential equations of first order, linear, with constant coefficients satisfying the initial conditions: $\mathrm{Y}($ year_1 $)=\mathrm{Y}_{0}, \mathrm{r}($ year_1 $)=\mathrm{r}_{0}$ where year_1 is the first year of analysis.

Let now the matrix of the system:
(70) $\mathrm{M}=\left(\begin{array}{c}\mathrm{A} \alpha \\ \mathrm{Bmd}_{\mathrm{Y}}\end{array}\right.$
$\left.\begin{array}{c}\mathrm{Ai}_{\mathrm{r}} \\ \mathrm{Bmd}_{\mathrm{r}}\end{array}\right)$
and the characteristic equation for eigenvalues determination:
(71)

$$
\left|\begin{array}{cc}
\operatorname{A\alpha -\lambda } & \mathrm{Ai}_{\mathrm{r}} \\
\operatorname{Bmd}_{\mathrm{Y}} & \operatorname{Bmd}_{\mathrm{r}}-\lambda
\end{array}\right|=0
$$

that is: $\lambda^{2}-\left(A \alpha+B m d_{r}\right) \lambda+A B\left(\alpha m d_{r}-i_{r} m d_{Y}\right)=0$. Let the discriminant of the equation:
(72) $\Delta=\left(\mathrm{A} \alpha-\mathrm{Bmd}_{\mathrm{r}}\right)^{2}+4 \mathrm{ABi}_{\mathrm{r}} \mathrm{md}_{\mathrm{Y}}$
and $\lambda_{1}, \lambda_{2} \in \mathbf{C}$ the eigenvalues.
For the beginning we must determine a particular solution of the system (94).
Case p. $1 \alpha_{\mathrm{r}} \mathrm{md}_{\mathrm{r}} \mathrm{md}_{\mathrm{Y}} \neq 0$
In this case a particular solution has the expression: $\left\{\begin{array}{l}\mathrm{Y}_{0}(\mathrm{t})=\mathrm{at}+\mathrm{b} \\ \mathrm{r}_{0}(\mathrm{t})=\mathrm{ct}+\mathrm{d}\end{array}\right.$ where $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d} \in \mathbf{R}$ are determined after replacing in the system (93).

Case p. $2 \alpha \mathrm{md}_{\mathrm{r}}-\mathrm{i}_{\mathrm{r}} \mathrm{md}_{\mathrm{Y}}=0, \mathrm{~A} \alpha+\mathrm{Bmd}_{\mathrm{r}} \neq 0$
In this case a particular solution has the expression: $\left\{\begin{array}{l}\mathrm{Y}_{0}(\mathrm{t})=\mathrm{t}(\mathrm{at}+\mathrm{b}) \\ \mathrm{r}_{0}(\mathrm{t})=\mathrm{t}(\mathrm{ct}+\mathrm{d})\end{array}\right.$ where a,b,c,d $\in \mathbf{R}$ are determined after replacing in the system (93).

Case p. $3 \alpha_{\mathrm{r}} \mathrm{md}_{\mathrm{r}}-\mathrm{i}_{\mathrm{r}} \mathrm{md}_{\mathrm{Y}}=0, \mathrm{~A} \alpha+\mathrm{Bmd}_{\mathrm{r}}=0$

In this case a particular solution has the expression: $\left\{\begin{array}{l}Y_{0}(t)=t^{2}(a t+b) \\ r_{0}(t)=t^{2}(c t+d)\end{array}\right.$ where $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d} \in \mathbf{R}$ are determined after replacing in the system (93).

After the determination of particular solution, we have the following cases for the solution of homogenous system: $\binom{\frac{d Y}{d t}}{\frac{d r}{d t}}=\left(\begin{array}{cc}A \alpha & {A i_{r}}^{A} \\ B_{\mathrm{Y}} & \mathrm{Bmd}_{\mathrm{r}}\end{array}\right)\binom{\mathrm{Y}(\mathrm{t})}{\mathrm{r}(\mathrm{t})}$.

## Case o.1 $\Delta>0\left(\lambda_{1} \neq \lambda_{2}\right)$

The solution is: $\left\{\begin{array}{l}Y_{\text {hom }}(t)=C_{1} e^{\lambda_{1} t}+C_{2} e^{\lambda_{2} t} \\ r_{\text {hom }}(t)=C_{3} e^{\lambda_{1} t}+C_{4} e^{\lambda_{2} t}\end{array}\right.$
where $\mathrm{C}_{1}, \mathrm{C}_{2}, \mathrm{C}_{3}, \mathrm{C}_{4} \in \mathbf{R}$ will be determined by replacing in the homogenous system.
Case o.2 $\Delta=0\left(\lambda_{1}=\lambda_{2}=\lambda\right)$
The solution is: $\left\{\begin{array}{l}Y_{\text {hom }}(t)=\left(C_{1} t+C_{2}\right) e^{\lambda t} \\ r_{\text {hom }}(t)=\left(C_{3} t+C_{4}\right) e^{\lambda t}\end{array}\right.$
where $\mathrm{C}_{1}, \mathrm{C}_{2}, \mathrm{C}_{3}, \mathrm{C}_{4} \in \mathbf{R}$ will be determined by replacing in the homogenous system.
Case o. $3 \Delta<0\left(\lambda_{1}=\alpha_{1}+\mathrm{i} \beta_{1}, \lambda_{2}=\alpha_{1}-\mathrm{i} \beta_{1}\right)$
The solution is: $\left\{\begin{array}{l}Y_{\text {hom }}(t)=C_{1} e^{\alpha_{1} t} \cos \beta_{1} t+C_{2} e^{\alpha_{1} t} \sin \beta_{1} t \\ r_{\text {hom }}(t)=C_{3} e^{\alpha_{1} t} \cos \beta_{1} t+C_{4} e^{\alpha_{1} t} \sin \beta_{1} t\end{array}\right.$
where $\mathrm{C}_{1}, \mathrm{C}_{2}, \mathrm{C}_{3}, \mathrm{C}_{4} \in \mathbf{R}$ will be determined by replacing in the homogenous system.
Finally the general solution will be:
(73) $\left\{\begin{array}{l}\mathrm{Y}(\mathrm{t})=\mathrm{Y}_{\text {hom }}(\mathrm{t})+\mathrm{Y}_{0}(\mathrm{t}) \\ \mathrm{r}(\mathrm{t})=\mathrm{r}_{\text {hom }}(\mathrm{t})+\mathrm{r}_{0}(\mathrm{t})\end{array}\right.$
which is dependent on two arbitrary constants. From the initial conditions: $\mathrm{Y}\left(\right.$ year_1) $=\mathrm{Y}_{0}$, $r($ year_1 $)=r_{0}$ there will be determined.

## 6 Application of the model to the Romanian economy

After the regression analysis we find:
(74) $\mathrm{C}_{\mathrm{V}}=1.062338107, \mathrm{C}_{0}=-21306.522399, \mathrm{i}_{\mathrm{G}}=0.281763291, \mathrm{i}_{\mathrm{OR}}=0.077131491$, $\mathrm{OR}_{0}=8586.917756, \mathrm{r}_{\mathrm{CH}}=0.112581319, \mathrm{CH}_{0}=-222.1708473, \mathrm{c}_{\mathrm{TF}}=0.353272369$, $\mathrm{TF}_{0}=-24079.51702, \mathrm{t}_{\mathrm{Y}}=0.395122134, \mathrm{TR}_{0}=-25436.01202, \mathrm{~m}_{\mathrm{S}}=2745.9441, \mathrm{M}_{0}=-$ 5471920.509, $\mathrm{i}_{\mathrm{Y}}=0.308842141$,
$\mathrm{i}_{\mathrm{r}}=-1301.197683, \mathrm{im}_{\mathrm{Y}}=2.468228803, \mathrm{C}_{\mathrm{e}}=-20686.68561, \mathrm{IM}_{0}=-117531.7752$, $\mathrm{ex}_{\mathrm{Y}}=0.970442258, \mathrm{c}_{\mathrm{ee}}=606.9387431, \mathrm{EX}_{0}=-57581.34747, \mathrm{md}_{\mathrm{Y}}=0.416549399$, $\mathrm{md}_{\mathrm{r}}=-2860.243226$,
$\alpha=-1.03800116, \beta=-2397.26431, \gamma=-4695485.06$

Computing now for the equilibrium the values of GDP and of Real Interest Rate for each year from the period and after replacing in (28)-(40) we find the following situations.

Table 1

| Year | GDP real - Y | GDP for equilibrium - Y* |
| :---: | :---: | :---: |
| $\mathbf{2 0 0 1}$ | 85820.2 | 91058.11 |
| $\mathbf{2 0 0 2}$ | 90269.27724 | 94028.75 |
| $\mathbf{2 0 0 3}$ | 95255.99981 | 96999.39 |
| $\mathbf{2 0 0 4}$ | 103218.094 | 99970.04 |
| $\mathbf{2 0 0 5}$ | 107524.2492 | 102940.7 |
| $\mathbf{2 0 0 6}$ | 116185.9298 | 105911.3 |
| $\mathbf{2 0 0 7}$ | 124160.639 | 108882 |
| $\mathbf{2 0 0 8}$ | 134663.3768 | 111852.6 |
| $\mathbf{2 0 0 9}$ | 125146.9341 | 114823.2 |
| $\mathbf{2 0 1 0}$ | 124147.6909 | 117793.9 |
| $\mathbf{2 0 1 1}$ | 125459.0429 | 120764.5 |
| $\mathbf{2 0 1 2}$ | 126263.2192 | 123735.2 |
| $\mathbf{2 0 1 3}$ | 130722.3328 | 126705.8 |
| $\mathbf{2 0 1 4}$ | 134590.4634 | 129676.5 |



Figure 1
The analysis of GDP growth in the analyzed period reflects different situations. Thus, during 2001-2003, real GDP level was below the equilibrium, which somehow justified by the relocation of economy to one capitalist after the complicated decade at the end of the century

The second period 2004-2009, especially after 2005, was under the influence of liberal policies to stimulate consumption which led to a disproportionate rise in GDP far above the real possibilities of the Romanian economy. As we will see below, consumption growth was made, in particular based on massive imports, a lending to households with no sense and safety rule. Enlightening this is 2008 when the world economy into recession started and consumption in Romania reached paroxysmal.

After 2009, the real GDP starts to approach the balance, although still high, beaing clearly influenced by the strong economic crisis that affected Romania.

Table 2

| Year | Actual final consumption of <br> households real - C | Actual final consumption of households <br> for equilibrium - C* |
| :---: | :---: | :---: |
| $\mathbf{2 0 0 1}$ | 67758.83 | 72820.71 |
| $\mathbf{2 0 0 2}$ | 70876.21 | 75844.47 |
| $\mathbf{2 0 0 3}$ | 74269.63 | 78868.23 |
| $\mathbf{2 0 0 4}$ | 83028.18 | 81891.98 |
| $\mathbf{2 0 0 5}$ | 92658.84 | 84915.74 |
| $\mathbf{2 0 0 6}$ | 103566.1 | 87939.5 |
| $\mathbf{2 0 0 7}$ | 137896.7 | 90963.25 |
| $\mathbf{2 0 0 8}$ | 156482.3 | 93987.01 |
| $\mathbf{2 0 0 9}$ | 107423 | 97010.77 |
| $\mathbf{2 0 1 0}$ | 109358.3 | 100034.5 |
| $\mathbf{2 0 1 1}$ | 116227.7 | 103058.3 |
| $\mathbf{2 0 1 2}$ | 121122.3 | 106082 |
| $\mathbf{2 0 1 3}$ | 112366.5 | 109105.8 |
| $\mathbf{2 0 1 4}$ | 117094.6 | 112129.5 |



Figure 2

Table 3

| Year | Disposable Income real - DI | Disposable Income for <br> equilibrium - DI* |
| :---: | :---: | :---: |
| $\mathbf{2 0 0 1}$ | 84098.61 | 88603.84 |
| $\mathbf{2 0 0 2}$ | 88546.04 | 91450.16 |
| $\mathbf{2 0 0 3}$ | 92573.44 | 94296.49 |
| $\mathbf{2 0 0 4}$ | 97305.46 | 97142.81 |
| $\mathbf{2 0 0 5}$ | 106691.5 | 99989.13 |
| $\mathbf{2 0 0 6}$ | 113882.8 | 102835.5 |
| $\mathbf{2 0 0 7}$ | 148041.2 | 105681.8 |
| $\mathbf{2 0 0 8}$ | 170114.5 | 108528.1 |
| $\mathbf{2 0 0 9}$ | 121612.2 | 111374.4 |
| $\mathbf{2 0 1 0}$ | 123329.5 | 114220.7 |
| $\mathbf{2 0 1 1}$ | 127768.7 | 117067.1 |
| $\mathbf{2 0 1 2}$ | 130399.3 | 119913.4 |
| $\mathbf{2 0 1 3}$ | 127005.3 | 122759.7 |
| $\mathbf{2 0 1 4}$ | 133280.9 | 125606 |



Figure 3
Analysis of household consumption and disposable income reflects an apparently paradoxical. First, one should note the marginal propensity to consume $\mathrm{c}_{\mathrm{V}}=1.06$ whose value (indeed, statistically determined relative to the entire period) exceeds theoretical considerations, normally at odds subunit. Although the regression equation for 2010-2014 recalculated marginal propensity to consume to 0.967 it remains extraordinarily high.

Comparative analysis of the evolution of Disposable Income and Actual final consumption of households during 2001-2014 reflects a share of consumption in Disposable Income between $80 \%$ (2002-2003) and over 93\% (2007) (figure 4). If until the year 2003 the situation can be understood against the background of an adaptation of the consumption needs of the modern world, after this year it is again symptomatic of the mess Romanian economy. In 2007, the share of $93.15 \%$ in Disposable Income related to consumer credit expansion to households shows an endowment hysteria especially consumer goods far beyond the common man. The emergence of the economic crisis has tempered the phenomenon to a very small extent, even if Disposable Income decreased, consumption share remained very high, hovering somewhere at $88-93 \%$. This reflects low economic education of the Romanian population, justified with distrust in the future, due to possible inflation that actual savings may decrease.

Analysis of changes in consumption during 2007-2014, deposits and loans, even if statistical provides a very low correlation between them, reveals an increase of $13.48 \%$ in 2008 to a growth of consumption loans of the population $-28.53 \%$. Also, a paradoxical situation was in 2011 when credit was reduced from the previous year to $3.09 \%$, disposable income increased by $3.60 \%$, but consumption was increased by $6.28 \%$ while only declarative Romania was out of the crisis. Again in 2013, the disposable income fell by $2.60 \%$ and consumption by $7.23 \%$. This situation can be explained easily by decreasing $4.25 \%$ crediting. As a conclusion, it emerges that the evolution of consumption is dependent simultaneously from the change in disposable income and household lending.


Figure 4


Figure 5

Table 4

| Year | The actual final consumption <br> of the government real - G | The actual final consumption of the <br> government for equilibrium - G* |
| :---: | :---: | :---: |
| $\mathbf{2 0 0 1}$ | 6288.275 | 7369.076 |
| $\mathbf{2 0 0 2}$ | 6198.451 | 7764.36 |
| $\mathbf{2 0 0 3}$ | 9655.797 | 8159.645 |
| $\mathbf{2 0 0 4}$ | 8478.723 | 8554.93 |
| $\mathbf{2 0 0 5}$ | 10846.86 | 8950.215 |
| $\mathbf{2 0 0 6}$ | 10089.7 | 9345.5 |
| $\mathbf{2 0 0 7}$ | 13961.99 | 9740.785 |
| $\mathbf{2 0 0 8}$ | 16345.71 | 10136.07 |
| $\mathbf{2 0 0 9}$ | 11696.16 | 10531.35 |
| $\mathbf{2 0 1 0}$ | 10681.44 | 10926.64 |
| $\mathbf{2 0 1 1}$ | 10181.21 | 11321.92 |
| $\mathbf{2 0 1 2}$ | 10559.68 | 11717.21 |
| $\mathbf{2 0 1 3}$ | 11511.67 | 12112.49 |
| $\mathbf{2 0 1 4}$ | 12897.15 | 12507.78 |



Figure 6
Analysis of government consumption proves again irresponsibility in public spending. Thus, in 2007-2009 they amounted to enormous value to the equilibrium level even in the last part of the period when there were obvious signs of the economic crisis. With the austerity measures taken in 2010, they fell far below the equilibrium level which meant a restructuring of the bureaucracy, but not enough.

## Table 5

| Year | Real Interest Rate - r(\%) | Real Interest Rate for <br> equilibrium - (* (\%) |
| :---: | :---: | :---: |
| $\mathbf{2 0 0 1}$ | 3.2 | 5.320025 |
| $\mathbf{2 0 0 2}$ | 4.87 | 4.792614 |
| $\mathbf{2 0 0 3}$ | 3.44 | 4.265202 |
| $\mathbf{2 0 0 4}$ | 7.09 | 3.737791 |
| $\mathbf{2 0 0 5}$ | 1.99 | 3.21038 |
| $\mathbf{2 0 0 6}$ | 1.84 | 2.682968 |
| $\mathbf{2 0 0 7}$ | 2.4 | 2.155557 |
| $\mathbf{2 0 0 8}$ | 1.71 | 1.628146 |
| $\mathbf{2 0 0 9}$ | 3.28 | 1.100734 |
| $\mathbf{2 0 1 0}$ | 0.34 | 0.573323 |
| $\mathbf{2 0 1 1}$ | 0.39 | 0.045912 |
| $\mathbf{2 0 1 2}$ | 1.85 | -0.4815 |
| $\mathbf{2 0 1 3}$ | 1.56 | -1.00891 |
| $\mathbf{2 0 1 4}$ | 1.88 | -1.53632 |



Figure 7
Relative to the real interest rate in the period analyzed it oscillated around balance. But here are a series of interesting issues. In 2004 it was located at a value more than 3 percentage points higher than the optimal (in terms of mathematical model), which however has not been seen in the fall in investments, which are very close to the equilibrium level. The next period, 2005-2006 represented a decrease of approximately 1.5 percentage points below the equilibrium rate which was reflected in credit growth with negative effects on consumption that we have analyzed above. In 2010-2011, the real rate ranged around equilibrium which contributed to an increase in real investment. Since 2012, the real rate was again over the balance, having suffered again investments especially in the post-crisis period that would have to generate a new impetus to the Romanian economy.

Table 6

| Year | Investments real - I | Investments for equilibrium - I* |
| :---: | :---: | :---: |
| $\mathbf{2 0 0 1}$ | 18663.1 | 21200.18 |
| $\mathbf{2 0 0 2}$ | 19493.34 | 22803.9 |
| $\mathbf{2 0 0 3}$ | 21214.23 | 24407.63 |
| $\mathbf{2 0 0 4}$ | 26643.54 | 26011.35 |
| $\mathbf{2 0 0 5}$ | 25779.78 | 27615.08 |
| $\mathbf{2 0 0 6}$ | 35233.9 | 29218.81 |
| $\mathbf{2 0 0 7}$ | 36134.58 | 30822.53 |
| $\mathbf{2 0 0 8}$ | 32262.35 | 32426.26 |
| $\mathbf{2 0 0 9}$ | 39412.95 | 34029.98 |
| $\mathbf{2 0 1 0}$ | 40779.26 | 35633.71 |
| $\mathbf{2 0 1 1}$ | 40235.91 | 37237.44 |
| $\mathbf{2 0 1 2}$ | 34310.17 | 38841.16 |
| $\mathbf{2 0 1 3}$ | 40352.81 | 40444.89 |
| $\mathbf{2 0 1 4}$ | 42033.73 | 42048.62 |



Figure 8
Increasing investment would seem, at first glance, favorable, hovering mostly above the equilibrium level. From Figure 8 it can be seen easily that social-democratic regimes of the periods 2001-2004, 2012-2014 respectively, placed investments suboptimal what was seen, especially in the first period, to the suboptimal situation of GDP. In the period 2006-2011 investments known, at least in value terms, a very large scale, but the problem is that about their quality and direction and not about their volume. Investments in infrastructure which claimed huge costs without finality, referring to roads, or capacities that subsequently were exploited, example of hospitals constructed, equipped and then dismantled, prove an investment activity without clearly outlined direction.

Table 7

| Year | Exports real - EX | Exports for equilibrium - EX* |
| :---: | :---: | :---: |
| $\mathbf{2 0 0 1}$ | 29755.6 | 32669.45 |
| $\mathbf{2 0 0 2}$ | 34785.2 | 35620.62 |
| $\mathbf{2 0 0 3}$ | 38135.36 | 38571.78 |
| $\mathbf{2 0 0 4}$ | 42965.73 | 41522.95 |
| $\mathbf{2 0 0 5}$ | 48012.61 | 44474.12 |
| $\mathbf{2 0 0 6}$ | 53281.08 | 47425.29 |
| $\mathbf{2 0 0 7}$ | 68858.56 | 50376.45 |
| $\mathbf{2 0 0 8}$ | 70901.38 | 53327.62 |
| $\mathbf{2 0 0 9}$ | 49910.08 | 56278.79 |
| $\mathbf{2 0 1 0}$ | 58177.35 | 59229.96 |
| $\mathbf{2 0 1 1}$ | 68029.9 | 62181.12 |
| $\mathbf{2 0 1 2}$ | 70800.84 | 65132.29 |
| $\mathbf{2 0 1 3}$ | 79950.3 | 68083.46 |
| $\mathbf{2 0 1 4}$ | 88336.05 | 71034.62 |



Figure 9
Table 8

| Year | Imports real - IM | Imports for equilibrium - IM* |
| :---: | :---: | :---: |
| $\mathbf{2 0 0 1}$ | 36645.6 | 43001.31 |
| $\mathbf{2 0 0 2}$ | 41083.92 | 48004.6 |
| $\mathbf{2 0 0 3}$ | 48019.02 | 53007.89 |
| $\mathbf{2 0 0 4}$ | 57898.08 | 58011.19 |
| $\mathbf{2 0 0 5}$ | 69773.84 | 63014.48 |
| $\mathbf{2 0 0 6}$ | 85984.83 | 68017.77 |
| $\mathbf{2 0 0 7}$ | 132691.2 | 73021.06 |
| $\mathbf{2 0 0 8}$ | 141328.4 | 78024.35 |
| $\mathbf{2 0 0 9}$ | 83295.29 | 83027.64 |
| $\mathbf{2 0 1 0}$ | 94848.69 | 88030.94 |
| $\mathbf{2 0 1 1}$ | 109215.6 | 93034.23 |
| $\mathbf{2 0 1 2}$ | 110529.8 | 98037.52 |
| $\mathbf{2 0 1 3}$ | 113459 | 103040.8 |
| $\mathbf{2 0 1 4}$ | 125771.1 | 108044.1 |



Figure 10
Table 9

| Year | Net Exports real - NX | Net Exports for equilibrium - NX* |
| :---: | :---: | :---: |
| $\mathbf{2 0 0 1}$ | -6890 | -10331.9 |
| $\mathbf{2 0 0 2}$ | -6298.72 | -12384 |
| $\mathbf{2 0 0 3}$ | -9883.66 | -14436.1 |
| $\mathbf{2 0 0 4}$ | -14932.3 | -16488.2 |
| $\mathbf{2 0 0 5}$ | -21761.2 | -18540.4 |
| $\mathbf{2 0 0 6}$ | -32703.7 | -20592.5 |
| $\mathbf{2 0 0 7}$ | -63832.7 | -22644.6 |
| $\mathbf{2 0 0 8}$ | -70427 | -24696.7 |
| $\mathbf{2 0 0 9}$ | -33385.2 | -26748.9 |
| $\mathbf{2 0 1 0}$ | -36671.3 | -28801 |
| $\mathbf{2 0 1 1}$ | -41185.7 | -30853.1 |
| $\mathbf{2 0 1 2}$ | -39728.9 | -32905.2 |
| $\mathbf{2 0 1 3}$ | -33508.7 | -34957.4 |
| $\mathbf{2 0 1 4}$ | -37435 | -37009.5 |



Figure 11
Romania's foreign trade evolution best explains the need for radical restructuring of the national economy. From Figure 11 we can see that if, in real terms, in 2001-2004 even though the trade balance was negative, overall net exports was above the equilibrium, in the next period, the so-called boom has been a disaster . Stimulating consumption was done almost exclusively on imports, which were in 2007-2008 almost two times higher than the equilibrium level. Lack of domestic production capacities, referring specifically to consumer durables, consumer loans with bulletin led to a massive demand from people for import products which led to a huge imbalance in the trade balance. The beginning of the economic crisis tempered enthusiasm and the gap was reduced. Forecast model of balance is negative, however, the current state of the Romanian economy by emphasizing leading trade deficit.

Table 10

| Year | Total Income real - TI | Total Income for equilibrium - TI* |
| :---: | :---: | :---: |
| $\mathbf{2 0 0 1}$ | 25511.03 | 26153.43 |
| $\mathbf{2 0 0 2}$ | 26506.91 | 27556.32 |
| $\mathbf{2 0 0 3}$ | 28050.69 | 28959.22 |
| $\mathbf{2 0 0 4}$ | 30400.21 | 30362.12 |
| $\mathbf{2 0 0 5}$ | 33335.19 | 31765.01 |
| $\mathbf{2 0 0 6}$ | 36957.62 | 33167.91 |
| $\mathbf{2 0 0 7}$ | 46694.96 | 34570.81 |
| $\mathbf{2 0 0 8}$ | 55937.3 | 35973.7 |
| $\mathbf{2 0 0 9}$ | 37707.38 | 37376.6 |
| $\mathbf{2 0 1 0}$ | 38814.42 | 38779.5 |
| $\mathbf{2 0 1 1}$ | 41771.27 | 40182.39 |
| $\mathbf{2 0 1 2}$ | 43657.57 | 41585.29 |
| $\mathbf{2 0 1 3}$ | 41321.81 | 42988.19 |
| $\mathbf{2 0 1 4}$ | 44115.83 | 44391.09 |



Figure 12
Total Income reveals a situation theoretically favorable, mostly they are above the balance. What should be noted is the fact that statistical analysis takes into account existing revenues and not those who could come through reducing tax evasion. However, it may be noted that in 2005-2008, due to lower tax rate to $16 \%$, the actual level of Total Income has greatly increased which contributed essentially to alleviate imbalances State Budget.

Table 11

| Year | Tax rate real - TR | Tax rate for equilibrium - TR* |
| :---: | :---: | :---: |
| $\mathbf{2 0 0 1}$ | 10565.3 | 10543.06 |
| $\mathbf{2 0 0 2}$ | 10604.57 | 11716.83 |
| $\mathbf{2 0 0 3}$ | 12205.86 | 12890.59 |
| $\mathbf{2 0 0 4}$ | 13314.69 | 14064.36 |
| $\mathbf{2 0 0 5}$ | 14054.55 | 15238.13 |
| $\mathbf{2 0 0 6}$ | 22057.47 | 16411.9 |
| $\mathbf{2 0 0 7}$ | 28054.03 | 17585.66 |
| $\mathbf{2 0 0 8}$ | 32048.22 | 18759.43 |
| $\mathbf{2 0 0 9}$ | 20882.52 | 19933.2 |
| $\mathbf{2 0 1 0}$ | 21399.06 | 21106.96 |
| $\mathbf{2 0 1 1}$ | 24037.53 | 22280.73 |
| $\mathbf{2 0 1 2}$ | 25777.67 | 23454.5 |
| $\mathbf{2 0 1 3}$ | 24563.21 | 24628.26 |
| $\mathbf{2 0 1 4}$ | 25783.26 | 25802.03 |



Figure 13
Tax Rate evolution follows essentially the same trend with the Total Income, largely above the equilibrium level, in 2005-2008, due to reduced tax rate to $16 \%$, the greatly increasing of actual Tax Rate being 56.94\% - from 2005 to 2006 and 27.19\% - 2007 versus 2006. The last years of analysis (2013-2014) again shows a sinuous evolution of this indicator, after 2013 when there was a decrease of $4.71 \%$ (on the background of legislative changes and introduction of additional taxes), in 2014 returning to a growth of $4.97 \%$.

Table 12

| Year | Fiscality rate real TR/Y (\%) | Fiscality rate for <br> equilibrium - TR*/Y* (\%) |
| :---: | :---: | :---: |
| $\mathbf{2 0 0 1}$ | $12.31 \%$ | $11.58 \%$ |
| $\mathbf{2 0 0 2}$ | $11.77 \%$ | $12.46 \%$ |
| $\mathbf{2 0 0 3}$ | $12.79 \%$ | $13.29 \%$ |
| $\mathbf{2 0 0 4}$ | $13.04 \%$ | $14.07 \%$ |
| $\mathbf{2 0 0 5}$ | $12.72 \%$ | $14.80 \%$ |
| $\mathbf{2 0 0 6}$ | $18.38 \%$ | $15.50 \%$ |
| $\mathbf{2 0 0 7}$ | $18.26 \%$ | $16.15 \%$ |
| $\mathbf{2 0 0 8}$ | $18.09 \%$ | $16.77 \%$ |
| $\mathbf{2 0 0 9}$ | $17.06 \%$ | $17.36 \%$ |
| $\mathbf{2 0 1 0}$ | $17.42 \%$ | $17.92 \%$ |
| $\mathbf{2 0 1 1}$ | $18.53 \%$ | $18.45 \%$ |
| $\mathbf{2 0 1 2}$ | $19.16 \%$ | $18.96 \%$ |
| $\mathbf{2 0 1 3}$ | $18.69 \%$ | $19.44 \%$ |
| $\mathbf{2 0 1 4}$ | $18.72 \%$ | $19.90 \%$ |



Figure 14
What should be noted is the evolution of Fiscality Rate (the ratio between Tax Rate and GDP). If during the social-democratic regime or those of transition (2002-2005, 2013-2014) it was below the balance, primarily due to overly high taxes that led to modest revenue, during the 2006-2008 fiscal development was a favorable one, leading to higher receipts to the State Budget, primarily due to flat tax level of $16 \%$.

Table 13

| Year | Other revenues real - OR | Other revenues for equilibrium - <br> OR* |
| :---: | :---: | :---: |
| $\mathbf{2 0 0 1}$ | 14945.72 | 15610.37 |
| $\mathbf{2 0 0 2}$ | 15902.34 | 15839.5 |
| $\mathbf{2 0 0 3}$ | 15844.83 | 16068.63 |
| $\mathbf{2 0 0 4}$ | 17085.52 | 16297.76 |
| $\mathbf{2 0 0 5}$ | 19280.64 | 16526.89 |
| $\mathbf{2 0 0 6}$ | 14900.15 | 16756.02 |
| $\mathbf{2 0 0 7}$ | 18640.92 | 16985.15 |
| $\mathbf{2 0 0 8}$ | 23889.08 | 17214.28 |
| $\mathbf{2 0 0 9}$ | 16824.86 | 17443.41 |
| $\mathbf{2 0 1 0}$ | 17415.36 | 17672.54 |
| $\mathbf{2 0 1 1}$ | 17733.74 | 17901.67 |
| $\mathbf{2 0 1 2}$ | 17879.9 | 18130.8 |
| $\mathbf{2 0 1 3}$ | 16758.59 | 18359.93 |
| $\mathbf{2 0 1 4}$ | 18332.57 | 18589.06 |



Figure 15
The evolution of Other Revenues indicator experienced a fluctuating trend which has a character more or less short term.

Table 14

| Year | Government transfers real - TF | Government transfers for <br> equilibrium - TF* |
| :---: | :---: | :---: |
| $\mathbf{2 0 0 1}$ | 8843.708 | 8088.796 |
| $\mathbf{2 0 0 2}$ | 9026.951 | 9138.242 |
| $\mathbf{2 0 0 3}$ | 9371.477 | 10187.69 |
| $\mathbf{2 0 0 4}$ | 8492.239 | 11237.13 |
| $\mathbf{2 0 0 5}$ | 10240.93 | 12286.58 |
| $\mathbf{2 0 0 6}$ | 15956.71 | 13336.03 |
| $\mathbf{2 0 0 7}$ | 22442.4 | 14385.47 |
| $\mathbf{2 0 0 8}$ | 24974.67 | 15434.92 |
| $\mathbf{2 0 0 9}$ | 20070.41 | 16484.36 |
| $\mathbf{2 0 1 0}$ | 21874.64 | 17533.81 |
| $\mathbf{2 0 1 1}$ | 22052.25 | 18583.26 |
| $\mathbf{2 0 1 2}$ | 21605.22 | 19632.7 |
| $\mathbf{2 0 1 3}$ | 20110.36 | 20682.15 |
| $\mathbf{2 0 1 4}$ | 21336.87 | 21731.59 |



Figure 16
The Government Transfers were generally located above the balance. Social policies in Romania have been known for having sinuous developments in political regimes that have succeeded at intervals of 4 years. Thus, if in the 2002-2005 period they were below equilibrium, in the next they experienced exaggerated levels reaching in 2008 more than $160 \%$ of the optimal level, leading to major imbalances. Since 2013 they are about on optimal line which gives hopes to rebalance the State Budget.

Table 15

| Year | Exchange rate real - CH | Exchange rate for equilibrium - CH* |
| :---: | :---: | :---: |
| $\mathbf{2 0 0 1}$ | 2.6012 | 3.104371 |
| $\mathbf{2 0 0 2}$ | 3.1241 | 3.216953 |
| $\mathbf{2 0 0 3}$ | 3.7559 | 3.329534 |
| $\mathbf{2 0 0 4}$ | 4.0523 | 3.442115 |
| $\mathbf{2 0 0 5}$ | 3.6234 | 3.554697 |
| $\mathbf{2 0 0 6}$ | 3.5245 | 3.667278 |
| $\mathbf{2 0 0 7}$ | 3.3373 | 3.779859 |
| $\mathbf{2 0 0 8}$ | 3.6827 | 3.892441 |
| $\mathbf{2 0 0 9}$ | 4.2373 | 4.005022 |
| $\mathbf{2 0 1 0}$ | 4.2099 | 4.117603 |
| $\mathbf{2 0 1 1}$ | 4.2379 | 4.230185 |
| $\mathbf{2 0 1 2}$ | 4.456 | 4.342766 |
| $\mathbf{2 0 1 3}$ | 4.419 | 4.455347 |
| $\mathbf{2 0 1 4}$ | 4.4446 | 4.567929 |



Figure 17
The exchange rate, which at first had a sinuous evolution (being extremely high in 20032004 and then in 2006-2007 far below the equilibrium level - that favored massive imports) recorded after Romania's EU integration a level usually located very close to the equilibrium, which proves a fair policy for determining it.

Table 16

| Year | Money Demand real - MD | Money Demand for <br> equilibrium - MD* |
| :---: | :---: | :---: |
| $\mathbf{2 0 0 1}$ | 19619.66 | 20531.79 |
| $\mathbf{2 0 0 2}$ | 22066.63 | 23893.57 |
| $\mathbf{2 0 0 3}$ | 22116.15 | 27479.31 |
| $\mathbf{2 0 0 4}$ | 26465.94 | 32304.44 |
| $\mathbf{2 0 0 5}$ | 32802.96 | 35606.69 |
| $\mathbf{2 0 0 6}$ | 38187.59 | 40723.24 |
| $\mathbf{2 0 0 7}$ | 54385.87 | 45553.62 |
| $\mathbf{2 0 0 8}$ | 58668.23 | 51437.06 |
| $\mathbf{2 0 0 9}$ | 45085.86 | 48981.51 |
| $\mathbf{2 0 1 0}$ | 45924.48 | 50073.8 |
| $\mathbf{2 0 1 1}$ | 48691.6 | 52128.57 |
| $\mathbf{2 0 1 2}$ | 50140.47 | 53972.07 |
| $\mathbf{2 0 1 3}$ | 49751.52 | 57338.04 |
| $\mathbf{2 0 1 4}$ | 53964.85 | 60457.83 |



Figure 18
Finally, the Money Demand was located almost consistently below the equilibrium level except for the period 2007-2008 when has been far above because, first, high liquidity compared with other periods.
The second part of the analysis concerns the sensitivity of the two basic indicators: GDP and the real interest rate depending on the model parameters. From formulas (44)-(89) with parameter values in (99), we get:
(75) $\frac{\partial \mathrm{Y}}{\partial \mathrm{c}_{\mathrm{V}}}=-4567728.0137+2318.7958 \mathrm{t}, \frac{\partial \mathrm{Y}}{\partial \mathrm{C}_{0}}=0.8147$,

$$
\frac{\partial \mathrm{Y}}{\partial \mathrm{i}_{\mathrm{G}}}=-2265615.3687+1142.8894 \mathrm{t}
$$

(76) $\frac{\partial \mathrm{Y}}{\partial \mathrm{i}_{\mathrm{OR}}}=-1343556.9951+681.8884 \mathrm{t}, \frac{\partial \mathrm{Y}}{\partial \mathrm{OR}_{0}}=0.2295$,

$$
\frac{\partial \mathrm{Y}}{\partial \mathrm{i}_{\mathrm{Y}}}=-4768389.0629+2420.0754 \mathrm{t}
$$

(77) $\frac{\partial \mathrm{Y}}{\partial \mathrm{i}_{\mathrm{r}}}=864.0896-0.4297 \mathrm{t}, \frac{\partial \mathrm{Y}}{\partial \mathrm{im}_{\mathrm{Y}}}=4768389.0629-2420.0754 \mathrm{t}$,

$$
\frac{\partial \mathrm{Y}}{\partial \mathrm{c}_{\mathrm{ei}}}=180.9946-0.0917 \mathrm{t}
$$

(78) $\frac{\partial \mathrm{Y}}{\partial \mathrm{IM}_{0}}=-0.8147, \frac{\partial \mathrm{Y}}{\partial \mathrm{ex}_{\mathrm{Y}}}=-4768389.0629+2420.0754 \mathrm{t}$,

$$
\frac{\partial \mathrm{Y}}{\partial \mathrm{c}_{\mathrm{ee}}}=-180.9946+0.0917 \mathrm{t}
$$

(79) $\frac{\partial \mathrm{Y}}{\partial \mathrm{EX}_{0}}=0.8147, \frac{\partial \mathrm{Y}}{\partial \mathrm{r}_{\mathrm{CH}}}=17347.1465 \mathrm{t}, \frac{\partial \mathrm{Y}}{\partial \mathrm{CH}_{0}}=17347.1465$,

$$
\frac{\partial \mathrm{Y}}{\partial \mathrm{c}_{\mathrm{TF}}}=-5065641.4105+2570.9383 \mathrm{t}
$$

(80) $\frac{\partial \mathrm{Y}}{\partial \mathrm{TF}_{0}}=0.8654, \frac{\partial \mathrm{Y}}{\partial \mathrm{t}_{\mathrm{Y}}}=3722084.4154-1889.0499 \mathrm{t}, \frac{\partial \mathrm{Y}}{\partial \mathrm{TR}_{0}}=-0.6359$
(81) $\frac{\partial \mathrm{Y}}{\partial \mathrm{md}_{\mathrm{Y}}}=2169261.9508-1100.9541 \mathrm{t}, \frac{\partial \mathrm{Y}}{\partial \mathrm{md}_{\mathrm{r}}}=-393.0964+0.1955 \mathrm{t}, \frac{\partial \mathrm{Y}}{\partial \mathrm{m}_{\mathrm{S}}}=0.3706 \mathrm{t}$
(82) $\frac{\partial \mathrm{Y}}{\partial \mathrm{M}_{0}}=0.3706$
(83) $\frac{\partial \mathrm{r}}{\partial \mathrm{c}_{\mathrm{V}}}=-665.2177+0.3377 \mathrm{t}, \frac{\partial \mathrm{r}}{\partial \mathrm{C}_{0}}=0.0001186, \frac{\partial \mathrm{r}}{\partial \mathrm{i}_{\mathrm{G}}}=-329.9512+0.1664 \mathrm{t}$
(84) $\frac{\partial \mathrm{r}}{\partial \mathrm{i}_{\mathrm{OR}}}=-195.6679+0.0993 \mathrm{t}, \frac{\partial \mathrm{r}}{\partial \mathrm{OR}_{0}}=0.00003343, \frac{\partial \mathrm{r}}{\partial \mathrm{i}_{\mathrm{Y}}}=-694.4408+0.3524 \mathrm{t}$
(85) $\frac{\partial \mathrm{r}}{\partial \mathrm{i}_{\mathrm{r}}}=0.1258-0.00006257 \mathrm{t}, \frac{\partial \mathrm{r}}{\partial \mathrm{im}_{\mathrm{Y}}}=694.4408-0.3524 \mathrm{t}$,

$$
\frac{\partial \mathrm{r}}{\partial \mathrm{c}_{\mathrm{ei}}}=0.02636-0.00001336 \mathrm{t}
$$

(86) $\frac{\partial \mathrm{r}}{\partial \mathrm{IM}_{0}}=-0.0001186, \frac{\partial \mathrm{r}}{\partial \mathrm{ex}_{\mathrm{Y}}}=-694.4408+0.3524 \mathrm{t}, \frac{\partial \mathrm{r}}{\partial \mathrm{c}_{\mathrm{ee}}}=-0.02636+0.00001336 \mathrm{t}$
(87) $\frac{\partial \mathrm{r}}{\partial \mathrm{EX}_{0}}=0.0001186, \frac{\partial \mathrm{r}}{\partial \mathrm{r}_{\mathrm{CH}}}=2.5263 \mathrm{t}, \frac{\partial \mathrm{r}}{\partial \mathrm{CH}_{0}}=2.5263, \frac{\partial \mathrm{r}}{\partial \mathrm{c}_{\mathrm{TF}}}=-737.7309+0.3744 \mathrm{t}$
(88) $\frac{\partial \mathrm{r}}{\partial \mathrm{TF}_{0}}=0.0001260, \frac{\partial \mathrm{r}}{\partial \mathrm{t}_{\mathrm{Y}}}=542.0630-0.2751 \mathrm{t}, \frac{\partial \mathrm{r}}{\partial \mathrm{TR}_{0}}=-0.00009261$,
(89) $\frac{\partial \mathrm{r}}{\partial \mathrm{md}_{\mathrm{Y}}}=-1730.4799+0.8783 \mathrm{t}, \frac{\partial \mathrm{r}}{\partial \mathrm{md}_{\mathrm{r}}}=0.3136-0.0001559 \mathrm{t}, \frac{\partial \mathrm{r}}{\partial \mathrm{m}_{\mathrm{S}}}=-0.0002956 \mathrm{t}$
(90) $\frac{\partial \mathrm{r}}{\partial \mathrm{M}_{0}}=-0.0002956$

For the last year of analysis - 2014 we obtain:
(91) $\frac{\partial \mathrm{Y}}{\partial \mathrm{c}_{\mathrm{V}}}=102326.7017, \quad \frac{\partial \mathrm{Y}}{\partial \mathrm{C}_{0}}=0.8147, \quad \frac{\partial \mathrm{Y}}{\partial \mathrm{i}_{\mathrm{G}}}=36163.8176, \quad \frac{\partial \mathrm{Y}}{\partial \mathrm{i}_{\mathrm{OR}}}=29766.2455$, $\frac{\partial \mathrm{Y}}{\partial \mathrm{OR}_{0}}=0.2295$
(92) $\frac{\partial \mathrm{Y}}{\partial \mathrm{i}_{\mathrm{Y}}}=105642.7380, \quad \frac{\partial \mathrm{Y}}{\partial \mathrm{i}_{\mathrm{r}}}=-1.2516, \quad \frac{\partial \mathrm{Y}}{\partial \mathrm{im}_{\mathrm{Y}}}=-105642.7380, \quad \frac{\partial \mathrm{Y}}{\partial \mathrm{c}_{\mathrm{ei}}}=-3.7213$, $\frac{\partial \mathrm{Y}}{\partial \mathrm{IM}_{0}}=-0.8147$
(93) $\frac{\partial \mathrm{Y}}{\partial \mathrm{ex}_{\mathrm{Y}}}=105642.7380, \frac{\partial \mathrm{Y}}{\partial \mathrm{c}_{\mathrm{ee}}}=3.7213, \frac{\partial \mathrm{Y}}{\partial \mathrm{EX}_{0}}=0.8147, \frac{\partial \mathrm{Y}}{\partial \mathrm{r}_{\mathrm{CH}}}=34937152.9507$,
(94) $\frac{\partial \mathrm{Y}}{\partial \mathrm{CH}_{0}}=17347.1465, \frac{\partial \mathrm{Y}}{\partial \mathrm{c}_{\mathrm{TF}}}=112228.3063, \frac{\partial \mathrm{Y}}{\partial \mathrm{TF}_{0}}=0.8654, \frac{\partial \mathrm{Y}}{\partial \mathrm{t}_{\mathrm{Y}}}=-82462.0608$
(95) $\frac{\partial \mathrm{Y}}{\partial \mathrm{TR}_{0}}=-0.6359, \quad \frac{\partial \mathrm{Y}}{\partial \mathrm{md}_{\mathrm{Y}}}=-48059.5792, \quad \frac{\partial \mathrm{Y}}{\partial \mathrm{md}_{\mathrm{r}}}=0.5694, \quad \frac{\partial \mathrm{Y}}{\partial \mathrm{m}_{\mathrm{S}}}=746.4114$, $\frac{\partial \mathrm{Y}}{\partial \mathrm{M}_{0}}=0.3706$
(96) $\frac{\partial \mathrm{r}}{\partial \mathrm{c}_{\mathrm{V}}}=14.9023, \frac{\partial \mathrm{r}}{\partial \mathrm{C}_{0}}=0.0001186, \frac{\partial \mathrm{r}}{\partial \mathrm{i}_{\mathrm{G}}}=5.2667, \frac{\partial \mathrm{r}}{\partial \mathrm{i}_{\mathrm{OR}}}=4.3350$,

$$
\frac{\partial \mathrm{r}}{\partial \mathrm{OR}_{0}}=0.00003343
$$

(97) $\frac{\partial \mathrm{r}}{\partial \mathrm{i}_{\mathrm{Y}}}=15.3852, \frac{\partial \mathrm{r}}{\partial \mathrm{i}_{\mathrm{r}}}=-0.0001823, \frac{\partial \mathrm{r}}{\partial \mathrm{im}_{\mathrm{Y}}}=-15.3852, \frac{\partial \mathrm{r}}{\partial \mathrm{c}_{\mathrm{ei}}}=-0.0005420$, $\frac{\partial \mathrm{r}}{\partial \mathrm{IM}_{0}}=-0.0001186$
(98) $\frac{\partial \mathrm{r}}{\partial \mathrm{ex}_{\mathrm{Y}}}=15.3852, \quad \frac{\partial \mathrm{r}}{\partial \mathrm{c}_{\mathrm{ee}}}=0.0005420, \quad \frac{\partial \mathrm{r}}{\partial \mathrm{EX}_{0}}=0.0001186, \quad \frac{\partial \mathrm{r}}{\partial \mathrm{r}_{\mathrm{CH}}}=5088.0463$, $\frac{\partial \mathrm{r}}{\partial \mathrm{CH}_{0}}=2.5263$
(99) $\frac{\partial \mathrm{r}}{\partial \mathrm{c}_{\mathrm{TF}}}=16.3443, \quad \frac{\partial \mathrm{r}}{\partial \mathrm{TF}_{0}}=0.0001260, \quad \frac{\partial \mathrm{r}}{\partial \mathrm{t}_{\mathrm{Y}}}=-12.0093, \quad \frac{\partial \mathrm{r}}{\partial \mathrm{TR}_{0}}=-0.00009261$,
$\frac{\partial \mathrm{r}}{\partial \mathrm{md}_{\mathrm{Y}}}=38.3384$
(100) $\frac{\partial \mathrm{r}}{\partial \mathrm{md}_{\mathrm{r}}}=-0.0004542, \frac{\partial \mathrm{r}}{\partial \mathrm{m}_{\mathrm{S}}}=-0.5954, \frac{\partial \mathrm{r}}{\partial \mathrm{M}_{0}}=-0.0002956$

From these values we can obtain the following conclusions:

- An increase of the marginal propensity to consume with 0.01 will give a GDP growth of 1023 mil. lei 2000, but also an increase of the real interest rate with an absolute value $0.15 \%$;
- An increase of the marginal index of final consumption of the government with 0.01 will give a GDP growth of 362 mil. lei 2000 and also a non significant increase of the real interest rate with an absolute value $0.05 \%$;
- An increase of the marginal index of other revenues with 0.01 will give a GDP growth of 298 mil. lei 2000 and also a non significant increase of the real interest rate with an absolute value $0.04 \%$;
- An increase of the rate of investments with 0.01 will give a GDP growth of 1056 mil. lei 2000, but also an increase of the real interest rate with an absolute value $0.15 \%$;
- An increase of the factor of influence on the investment rate with 100 will give a GDP decrease of 125 mil. lei 2000 and also a non significant decrease of the real interest rate with an absolute value $0.01 \%$;
- An increase of the rate of imports with 0.01 will give a GDP decrease of 1056 mil. lei 2000 and also a decrease of the real interest rate with an absolute value $0.15 \%$;
- An increase of the factor of imports influence on the exchange rate with 1000 will give a GDP decrease of 3721 mil. lei 2000 and also a significant decrease of the real interest rate with an absolute value $0.5 \%$;
- An increase of the rate of exports with 0.01 will give a GDP increase of 1056 mil. lei 2000 and also an increase of the real interest rate with an absolute value $0.15 \%$;
- An increase of the factor of exports influence on the exchange rate with 1000 will give a GDP increase of 3721 mil. lei 2000 and also a significant increase of the real interest rate with an absolute value $0.5 \%$;
- An increase of the marginal index of the exchange rate according to time with 0.0001 will give a GDP increase of 3494 mil. lei 2000 and also an increase of the real interest rate with an absolute value $0.5 \%$;
- An increase of the marginal index of government transfers according to the output with 0.01 will give a GDP increase of 1122 mil. lei 2000 and also an increase of the real interest rate with an absolute value $0.16 \%$;
- An increase of the marginal index of tax rate according to the output with 0.01 will give a GDP decrease of 825 mil . lei 2000 and also a decrease of the real interest rate with an absolute value 0.12 \%;
- An increase of the rate of money demand in the economy with 0.01 will give a GDP decrease of 481 mil. lei 2000 and also an increase of the real interest rate with an absolute value 0.38 \%;
- An increase of the factor of influencing the demand for currency from the interest rate with 1000 will give a GDP increase of 569 mil. lei 2000 and also a decrease of the real interest rate with an absolute value 0.4 \%;
- An increase of the marginal index of the money supply according to time with 10 will give a GDP increase of 7464 mil. lei 2000 and also a very high decrease of the real interest rate with an absolute value $5 \%$;

Now, for the dinamics model with $A=-0.14265, B=-0.00031$ computed at
averages of ratios $\frac{\frac{d Y}{d t}}{D(t)-Y(t)}, \frac{\frac{d r}{d t}}{\mathrm{MD}(\mathrm{t})-\mathrm{MS}(\mathrm{t})}$ in the given period we have:
$(101)\left\{\begin{aligned} \mathrm{Y}(\mathrm{t})= & 1062.64-0.5274 \mathrm{t}+\left(1.0507 \mathrm{C}_{1}-0.0001926 \mathrm{C}_{2}\right) \mathrm{e}^{0.852 \mathrm{t}}+ \\ & \left(-0.0507 \mathrm{C}_{1}+0.0001926 \mathrm{C}_{2}\right) \mathrm{e}^{0.182 \mathrm{t}} \\ \mathrm{r}(\mathrm{t})= & 2970.64 \mathrm{t}-5835602.49+\left(276.804 \mathrm{C}_{1}-0.0507 \mathrm{C}_{2}\right) \mathrm{e}^{0.852 \mathrm{t}}+ \\ & \left(-276.804 \mathrm{C}_{1}+1.0507 \mathrm{C}_{2}\right) \mathrm{e}^{0.182 \mathrm{t}}\end{aligned}\right.$
$\mathrm{C}_{1}, \mathrm{C}_{2} \in \mathbf{R}$

## 7 Conclusions

The model presented above shows a more flexibility in macroeconomic modeling, because it removes the common assumptions of constancy of variables. Thus, imports, exports, government consumption, transfers etc. are approached by their econometric dependence of GDP and other variables.

Romania's situation, presented in the case study, reveals a contradictory economic policy, in 2004-2008, the Romanian economy being overheated.

Recent years (2013-2014) approached the interest rate and GDP from equilibrium, which was reflected in an dynamic increased of investments.

For Romania, the analysis of marginal indicators proposes as directions for growth: the increase of investments, net exports, government consumption marginal, but also a diminishing of the marginal propensity to consume.

We can estimate a prognosis for 2015 in order to verify the validity of the model.
The value for equilibrium for GDP in 2015 is: 132647.1 lei 2000. Because in the last period (2008-2014) the ratio between the real GDP and that of the equilibrium was approximately constant - 104.55\% we obtain an estimated value: 138677 lei 2000. Because the cumulative deflator between 2000 and 2015 is 0.1940 we obtain a prognosis: $Y(2015)=714829.7$ lei. The real value (estimated at the beginning of 2016) is 712932 lei therefore an error: $0.27 \%$.

We can conclude after this that the model verify well the real data.

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