Ioan, Cătălin Angelo

Article

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An Equilibrium Model for the Romanian Economy

Cătălin Angelo IOAN[1], Gina IOAN[2]

Abstract: The model presented in this article is an adaptation of the IS-LM model for an open economy in which both the static aspects and dynamic ones are approached. The determination of marginal main indicators of GDP and interest rates, allow to identify problems and the directions of action to achieve economic equilibrium.

Keywords: equilibrium, GDP, investments, interest rate, consumption

1 Introduction

The economic equilibrium problem, has origins and manifestations lost in the mists of time.

Economic thinkers from different current and ideologies as François Quesnay, Léon Walras, Vilfredo Pareto, Alfred Marshall studied this problem.

In the XX century, John Maynard Keynes formulate a first economic equilibrium model for a closed economy without governmental sector.

Because the equilibrium problem bore controversies on economics, it get further researches, today being analyzed the fluctuations that accompany this process.

Within theory of economic equilibrium, a synthetic analysis it is the IS-LM model consisting of simultaneous equilibrium in two markets, money market and the goods and services in an autarkic economy.

Starting from Keynesian macroeconomic equilibrium, in 1937, Roy Harrod, James Meade and John Hicks tried to express mathematical majors relations of Keynes' theory (Hahn, F.H., 1977).

Subsequent developments of Alvin Hansen of 1949 and 1953 play an important role in systematizing known IS-LM model, in his book (Hansen A.H., 1959) in order to get the curve IS, Hansen calls the investment demand function of Keynes and the neoclassical paradigm and for the LL curve is the curve of points where supply and demand (Beaud M., Dostaler G., 1996).

The IS-LM model (King R. G., 1993; Lawn P. A., 2003; Martínez-García E., Vilán D., 2012; Romer D., 2000; Schmitt-Grohe S., Uribe M., 2002; Weerapana, A., 2003) was the basis for further researches – theoretical or empirical.

After Samuelson and Solow which include the original model of the Phillips curve (1960), Fleming Mundell and Fleming include balance of payments (1960 and 1962).

Also, Modigliani and Friedman use the consumption function (1954 and 1957) and Tobin includes the demand for money (1958).

[1] Danubius University of Galati, Department of Economics, catalin_angelo_ioan@univ-danubius.ro
[2] Danubius University of Galati, Department of Economics, ginaioan@univ-danubius.ro

Although economic literature that explores New Open Economy Macroeconomics (NOEM models) is not as rich as that of the closed economy model, it is a significant theoretical edifice for the current macroeconomic modeling: Bergin (Bergin P., 2004), Schmitt-Grohe and Uribe (Smith, R.P.; Zoega, G., 2009), Justiniano and Preston (Justiniano A., Preston B., 2008), (Justiniano A., Preston B., 2010), Martínez-García and Vilán (Martínez-García E., Vilán D., 2012).

The new approach enables researchers to explain the new changes that have occurred in the international macroeconomic environment.

In this paper we propose, based on ideological vision and studies of the most important researchers in the field to determine a model for an open economy, with applications on the Romanian case.

2 The model equations

The first equation of the model is the formula of the aggregate demand:

(1) $D(t)=C(t)+G(t)+I(t)+EX(t)-IM(t)$

where

- $D(t)$ – the aggregate demand at the moment t;
- $C(t)$ – the actual final consumption of households at the moment t;
- $G(t)$ – the actual final consumption of the government at the moment t;
- $I(t)$ – the investments at the moment t;
- $EX(t)$ – the exports at the moment t;
- $IM(t)$ – the imports at the moment t

A second equation relates the actual final consumption of households according to disposable income:

(2) $C(t)=c_V DI(t)+C_0$, $C_0 \in \mathbf{R}$, $c_V>0$

where

- $DI(t)$ – the disposable income at the moment t;
- c_V – the marginal propensity to consume, $c_V=\frac{dC}{dDI}>0$;
- C_0 is the intrinsic achieved autonomous consumption of households

(3) $G(t)=i_G TI(t)$, $i_G \in (0,1)$

where

- $TI(t)$ – the total income at the moment t;
- i_G – the marginal index of final consumption of the government according to total income

(4) $TI(t)=TR(t)+OR(t)$

where:

- $TR(t)$ – tax rate at the moment t;

- $OR(t)$ – other revenues at the moment t

(5) $OR(t)=i_{OR}Y(t)+OR_0,\ i_{OR}\in(0,1),\ OR_0\in\mathbf{R}$

where:

- $Y(t)$ – the output at the moment t;
- i_{OR} – the marginal index of other revenues according to the output;
- OR_0 – the autonomous other revenues

(6) $I(t)=i_YY(t)+i_rr(t),\ i_Y\in(0,1),\ i_r<0$

where:

- $I(t)$ – investments at the moment t;
- $r(t)$ – the real interest rate at the moment t;
- i_Y – the rate of investments;
- i_r – a factor of influence on the investment rate

(7) $IM(t)=im_YY(t)+c_{ei}CH(t)+IM_0,\ im_Y>0,\ c_{ei}<0,\ IM_0\in\mathbf{R}$

where:

- $CH(t)$ – the exchange rate of the national currency based on the euro at the moment t;
- im_Y – the rate of imports;
- c_{ei} – a factor of imports influence on the exchange rate
- IM_0 – the autonomous imports

(8) $EX(t)=ex_YY(t)+c_{ee}CH(t)+EX_0,\ ex_Y>0,\ c_{ee}>0,\ EX_0\in\mathbf{R}$

where:

- ex_Y – the rate of exports;
- c_{ee} – a factor of exports influence on the exchange rate
- EX_0 – the autonomous exports

(9) $CH(t)=r_{CH}t+CH_0,\ r_{CH},\ CH_0\in\mathbf{R}$

where:

- r_{CH} – the marginal index of the exchange rate according to time;
- CH_0 – the intercept of the regression

(10) $TF(t)=c_{TF}Y(t)+TF_0,\ c_{TF}\in(0,1),\ TF_0\in\mathbf{R}$

where:

- $TF(t)$ – the government transfers at the moment t;
- c_{TF} – the marginal index of government transfers according to the output;
- TF_0 – the autonomous government transfers

(11) $TR(t)=t_YY(t)+TR_0,\ t_Y\in(0,1),\ TR_0\in\mathbf{R}$

where:

- t_Y – the marginal index of tax rate according to the output;
- TR_0 – the intercept of the regression

(12) $DI(t)=Y(t)+TF(t)-TR(t)$

(13) $D(t)=Y(t)$ – the equation of equilibrium at the moment t

(14) $MD(t)=md_Y Y(t)+md_r r(t)$, $md_Y \in (0,1)$, $md_r<0$

where:

- $MD(t)$ – the money demand in the economy at the moment t;
- md_Y – the rate of money demand in the economy;
- md_r – a factor of influencing the demand for currency from the interest rate

(15) $MS(t)=m_S t+M_0$, $m_M, M_0 \in \mathbf{R}$

where:

- $MS(t)$ – the money supply in the economy at the moment t;
- m_S – the marginal index of the money supply according to time;
- M_0 – the intercept of the regression

$MD(t)=MS(t)$ – the equation of equilibrium at the moment t.

3 The equilibrium at a fixed moment

From (4), (5), (11) we get:

(1) $TI(t)=(t_Y+i_{OR})Y(t)+TR_0+OR_0$

From (3), (17):

(2) $G(t)=(i_G t_Y+i_G i_{OR})Y(t)+i_G(TR_0+OR_0)$

From (7), (9):

(3) $IM(t)=im_Y Y(t)+c_{ei}r_{CH}t+c_{ei}CH_0+IM_0$

From (8), (9):

(4) $EX(t)=ex_Y Y(t)+c_{ee}r_{CH}t+c_{ee}CH_0+EX_0$

From (10), (11), (12) we get:

(5) $DI(t)=(1+c_{TF}-t_Y)Y(t)+TF_0-TR_0$

From (2), (21):

(6) $C(t)=(c_V+c_V c_{TF}-c_V t_Y)Y(t)+c_V(TF_0-TR_0)+C_0$

Now, from (1), (6), (18), (19), (20), (22) we have:

(7) $D(t)=(c_V+c_V c_{TF}-c_V t_Y+i_G t_Y+i_G i_{OR}+i_Y+ex_Y-im_Y)Y(t)+i_r r(t)+(c_{ee}r_{CH}-c_{ei}r_{CH})t+$
$c_V(TF_0-TR_0)+i_G(TR_0+OR_0)+(c_{ee}-c_{ei})CH_0+C_0+EX_0-IM_0$

From (13) and (23) we get the first equation of the equilibrium:

(8) $(c_V+c_Vc_{TF}-c_Vt_Y+i_Gt_Y+i_Gi_{OR}+i_Y+ex_Y-im_Y-1)Y(t)+i_rr(t)+(c_{ee}r_{CH}-c_{ei}r_{CH})t+$
$c_V(TF_0-TR_0)+i_G(TR_0+OR_0)+(c_{ee}-c_{ei})CH_0+C_0+EX_0-IM_0=0$

and from (14), (15), (16) we get the second equation of the equilibrium

(9) $md_YY(t)+md_rr(t)-m_St-M_0=0$

Let note now:

(10) $\alpha=c_V+c_Vc_{TF}-c_Vt_Y+i_Gt_Y+i_Gi_{OR}+i_Y+ex_Y-im_Y-1$

$\beta=c_{ei}r_{CH}-c_{ee}r_{CH}$

$\gamma=c_V(TF_0-TR_0)+i_G(TR_0+OR_0)+(c_{ee}-c_{ei})CH_0+C_0+EX_0-IM_0$

The equilibrium equations become:

(11) $$\begin{cases}\alpha Y(t)+i_rr(t)=\beta t-\gamma\\ md_YY(t)+md_rr(t)=m_St+M_0\end{cases}$$

The solutions of equilibrium are (noted with same symbols without being a confusion):

(12) $$\begin{cases}Y(t)=\dfrac{(\beta md_r-m_Si_r)}{\alpha md_r-md_Yi_r}t-\dfrac{(M_0i_r+\gamma md_r)}{\alpha md_r-md_Yi_r}\\ r(t)=\dfrac{(m_S\alpha-\beta md_Y)}{\alpha md_r-md_Yi_r}t+\dfrac{(\gamma md_Y+\alpha M_0)}{\alpha md_r-md_Yi_r}\end{cases}$$

At equilibrium, replacing (28) in (1)-(16), we have:

(13) $$C(t)=\frac{(\beta md_r-m_Si_r)+(\beta md_r-m_Si_r)c_{TF}-(\beta md_r-m_Si_r)t_Y}{\alpha md_r-md_Yi_r}c_Vt+\frac{(M_0i_r+\gamma md_r)t_Y-(M_0i_r+\gamma md_r)-(M_0i_r+\gamma md_r)c_{TF}}{\alpha md_r-md_Yi_r}c_V+c_V(TF_0-TR_0)+C_0$$

(14) $G(t)=i_GTI(t)=$
$$\frac{(\beta md_r-m_Si_r)t_Y+(\beta md_r-m_Si_r)i_{OR}}{\alpha md_r-md_Yi_r}i_Gt-\frac{(M_0i_r+\gamma md_r)t_Y+(M_0i_r+\gamma md_r)i_{OR}}{\alpha md_r-md_Yi_r}i_G+(TR_0+OR_0)i_G$$

(15) $TI(t)=$
$$\frac{(\beta md_r-m_Si_r)t_Y+(\beta md_r-m_Si_r)i_{OR}}{\alpha md_r-md_Yi_r}t-\frac{(M_0i_r+\gamma md_r)t_Y+(M_0i_r+\gamma md_r)i_{OR}}{\alpha md_r-md_Yi_r}+TR_0+OR_0$$

(16) $$OR(t)=i_{OR}Y(t)+OR_0=\frac{(\beta md_r-m_Si_r)i_{OR}}{\alpha md_r-md_Yi_r}t-\frac{(M_0i_r+\gamma md_r)i_{OR}}{\alpha md_r-md_Yi_r}+OR_0$$

(17) I(t)=
$$\frac{(\beta md_r - m_S i_r)i_Y + (m_S\alpha - \beta md_Y)i_r}{\alpha md_r - md_Y i_r}t + \frac{(\gamma md_Y + \alpha M_0)i_r - (M_0 i_r + \gamma md_r)i_Y}{\alpha md_r - md_Y i_r}$$

(18) IM(t)=
$$\frac{(\beta md_r - m_S i_r)im_Y + (\alpha md_r - md_Y i_r)c_{ei} r_{CH}}{\alpha md_r - md_Y i_r}t - \frac{(M_0 i_r + \gamma md_r)im_Y}{\alpha md_r - md_Y i_r} + c_{ei}CH_0 + IM_0$$

(19) EX(t)=
$$\frac{(\beta md_r - m_S i_r)ex_Y + (\alpha md_r - md_Y i_r)c_{ee} r_{CH}}{\alpha md_r - md_Y i_r}t - \frac{(M_0 i_r + \gamma md_r)ex_Y}{\alpha md_r - md_Y i_r} + c_{ee}CH_0 + EX_0$$

(20) $CH(t)=r_{CH}t+CH_0$

(21) $$TF(t)=\frac{(\beta md_r - m_S i_r)c_{TF}}{\alpha md_r - md_Y i_r}t - \frac{(M_0 i_r + \gamma md_r)c_{TF}}{\alpha md_r - md_Y i_r} + TF_0$$

(22) $$TR(t)=\frac{(\beta md_r - m_S i_r)t_Y}{\alpha md_r - md_Y i_r}t - \frac{(M_0 i_r + \gamma md_r)t_Y}{\alpha md_r - md_Y i_r} + TR_0$$

(23) $$DI(t)=\frac{(\beta md_r - m_S i_r) + (\beta md_r - m_S i_r)c_{TF} - (\beta md_r - m_S i_r)t_Y}{\alpha md_r - md_Y i_r}t + \frac{(M_0 i_r + \gamma md_r)t_Y - (M_0 i_r + \gamma md_r) - (M_0 i_r + \gamma md_r)c_{TF}}{\alpha md_r - md_Y i_r} + TF_0 - TR_0$$

$MD(t)=MS(t)=m_S t+M_0$.

4 The variations of equilibrium output and real interest rate based on the parameter values

First of all, we will compute the derivatives of functions α, β and γ in function of the parameters of the model.

(16) $$\frac{\partial\alpha}{\partial c_V}=1+c_{TF}-t_Y,\ \frac{\partial\alpha}{\partial C_0}=0,\ \frac{\partial\alpha}{\partial i_G}=t_Y+i_{OR},\ \frac{\partial\alpha}{\partial i_{OR}}=i_G,\ \frac{\partial\alpha}{\partial OR_0}=0,\ \frac{\partial\alpha}{\partial i_Y}=1,$$
$$\frac{\partial\alpha}{\partial i_r}=0,$$
$$\frac{\partial\alpha}{\partial im_Y}=-1,\ \frac{\partial\alpha}{\partial c_{ei}}=0,\ \frac{\partial\alpha}{\partial IM_0}=0,\ \frac{\partial\alpha}{\partial ex_Y}=1,\ \frac{\partial\alpha}{\partial c_{ee}}=0,\ \frac{\partial\alpha}{\partial EX_0}=0,\ \frac{\partial\alpha}{\partial r_{CH}}=0,$$
$$\frac{\partial\alpha}{\partial CH_0}=0,\ \frac{\partial\alpha}{\partial c_{TF}}=c_V,\ \frac{\partial\alpha}{\partial TF_0}=0,\ \frac{\partial\alpha}{\partial t_Y}=i_G-c_V,\ \frac{\partial\alpha}{\partial TR_0}=0,\ \frac{\partial\alpha}{\partial md_Y}=0,$$
$$\frac{\partial\alpha}{\partial md_r}=0,\ \frac{\partial\alpha}{\partial m_S}=0,\ \frac{\partial\alpha}{\partial M_0}=0$$

(17) $\frac{\partial\beta}{\partial c_V}=0,\ \frac{\partial\beta}{\partial C_0}=0,\ \frac{\partial\beta}{\partial i_G}=0,\ \frac{\partial\beta}{\partial i_{OR}}=0,\ \frac{\partial\beta}{\partial OR_0}=0,\ \frac{\partial\beta}{\partial i_Y}=0,\ \frac{\partial\beta}{\partial i_r}=0,\ \frac{\partial\beta}{\partial im_Y}=0,$

$\frac{\partial\beta}{\partial c_{ei}}=r_{CH},\ \frac{\partial\beta}{\partial IM_0}=0,\ \frac{\partial\beta}{\partial ex_Y}=0,\ \frac{\partial\beta}{\partial c_{ee}}=-r_{CH},\ \frac{\partial\beta}{\partial EX_0}=0,\ \frac{\partial\beta}{\partial r_{CH}}=c_{ei}-c_{ee},$

$\frac{\partial\beta}{\partial CH_0}=0,\ \frac{\partial\beta}{\partial c_{TF}}=0,\ \frac{\partial\beta}{\partial TF_0}=0,\ \frac{\partial\beta}{\partial t_Y}=0,\ \frac{\partial\beta}{\partial TR_0}=0,\ \frac{\partial\beta}{\partial md_Y}=0,\ \frac{\partial\beta}{\partial md_r}=0,$

$\frac{\partial\beta}{\partial m_S}=0,\ \frac{\partial\beta}{\partial M_0}=0$

(18) $\frac{\partial\gamma}{\partial c_V}=TF_0-TR_0,\ \frac{\partial\gamma}{\partial C_0}=1,\ \frac{\partial\gamma}{\partial i_G}=TR_0+OR_0,\ \frac{\partial\gamma}{\partial i_{OR}}=0,\ \frac{\partial\gamma}{\partial OR_0}=i_G,$

$\frac{\partial\gamma}{\partial i_Y}=0,\ \frac{\partial\gamma}{\partial i_r}=0,\ \frac{\partial\gamma}{\partial im_Y}=0,\ \frac{\partial\gamma}{\partial c_{ei}}=-CH_0,\ \frac{\partial\gamma}{\partial IM_0}=-1,\ \frac{\partial\gamma}{\partial ex_Y}=0,$

$\frac{\partial\gamma}{\partial c_{ee}}=CH_0,\ \frac{\partial\gamma}{\partial EX_0}=1,\ \frac{\partial\gamma}{\partial r_{CH}}=0,\ \frac{\partial\gamma}{\partial CH_0}=c_{ee}-c_{ei},\ \frac{\partial\gamma}{\partial c_{TF}}=0,\ \frac{\partial\gamma}{\partial TF_0}=c_V,$

$\frac{\partial\gamma}{\partial t_Y}=0,$

$\frac{\partial\gamma}{\partial TR_0}=-c_V,\ \frac{\partial\gamma}{\partial md_Y}=0,\ \frac{\partial\gamma}{\partial md_r}=0,\ \frac{\partial\gamma}{\partial m_S}=0,\ \frac{\partial\gamma}{\partial M_0}=0$

We have now:

(19)
$$\frac{\partial Y}{\partial c_V}=-\frac{(\beta md_r-m_Si_r)(1+c_{TF}-t_Y)}{(\alpha md_r-md_Yi_r)^2}md_rt+ md_r\left(\frac{TF_0-TR_0}{\alpha md_r-md_Yi_r}-\frac{(M_0i_r+\gamma md_r)(1+c_{TF}-t_Y)}{(\alpha md_r-md_Yi_r)^2}\right)$$

(20)
$$\frac{\partial Y}{\partial C_0}=-\frac{md_r}{\alpha md_r-md_Yi_r}$$

(21)
$$\frac{\partial Y}{\partial i_G}=-\frac{(\beta md_r-m_Si_r)(t_Y+i_{OR})}{(\alpha md_r-md_Yi_r)^2}md_rt+ md_r\left(\frac{TR_0+OR_0}{\alpha md_r-md_Yi_r}-\frac{(M_0i_r+\gamma md_r)(t_Y+i_{OR})}{(\alpha md_r-md_Yi_r)^2}\right)$$

(22)
$$\frac{\partial Y}{\partial i_{OR}}=-\frac{\beta md_r-m_Si_r}{(\alpha md_r-md_Yi_r)^2}i_Gmd_rt+\frac{M_0i_r+\gamma md_r}{(\alpha md_r-md_Yi_r)^2}i_Gmd_r$$

(23)
$$\frac{\partial Y}{\partial OR_0}=-\frac{i_Gmd_r}{\alpha md_r-md_Yi_r}$$

(24)
$$\frac{\partial Y}{\partial i_Y}=-\frac{(\beta md_r-m_Si_r)}{(\alpha md_r-md_Yi_r)^2}md_rt+\frac{(M_0i_r+\gamma md_r)}{(\alpha md_r-md_Yi_r)^2}md_r$$

(25) $$\frac{\partial Y}{\partial i_r}=\frac{\beta md_Y-\alpha m_S}{(\alpha md_r-md_Y i_r)^2}md_r t-\frac{\gamma md_Y+\alpha M_0}{(\alpha md_r-md_Y i_r)^2}md_r$$

(26) $$\frac{\partial Y}{\partial im_Y}=\frac{(\beta md_r-m_S i_r)}{(\alpha md_r-md_Y i_r)^2}md_r t-\frac{(M_0 i_r+\gamma md_r)}{(\alpha md_r-md_Y i_r)^2}md_r$$

(27) $$\frac{\partial Y}{\partial c_{ei}}=\frac{r_{CH}md_r}{\alpha md_r-md_Y i_r}t+\frac{md_r CH_0}{\alpha md_r-md_Y i_r}$$

(28) $$\frac{\partial Y}{\partial IM_0}=-\frac{md_r}{\alpha md_r-md_Y i_r}$$

(29) $$\frac{\partial Y}{\partial ex_Y}=-\frac{(\beta md_r-m_S i_r)}{(\alpha md_r-md_Y i_r)^2}md_r t+\frac{(M_0 i_r+\gamma md_r)}{(\alpha md_r-md_Y i_r)^2}md_r$$

(30) $$\frac{\partial Y}{\partial c_{ee}}=-\frac{r_{CH}md_r}{\alpha md_r-md_Y i_r}t-\frac{md_r CH_0}{\alpha md_r-md_Y i_r}$$

(31) $$\frac{\partial Y}{\partial EX_0}=-\frac{md_r}{\alpha md_r-md_Y i_r}$$

(32) $$\frac{\partial Y}{\partial r_{CH}}=\frac{md_r(c_{ei}-c_{ee})}{\alpha md_r-md_Y i_r}t$$

(33) $$\frac{\partial Y}{\partial CH_0}=-\frac{md_r(c_{ee}-c_{ei})}{\alpha md_r-md_Y i_r}$$

(34) $$\frac{\partial Y}{\partial c_{TF}}=-\frac{(\beta md_r-m_S i_r)}{(\alpha md_r-md_Y i_r)^2}c_V md_r t+\frac{(M_0 i_r+\gamma md_r)}{(\alpha md_r-md_Y i_r)^2}c_V md_r$$

(35) $$\frac{\partial Y}{\partial TF_0}=-\frac{c_V md_r}{\alpha md_r-md_Y i_r}$$

(36) $$\frac{\partial Y}{\partial t_Y}=-\frac{(\beta md_r-m_S i_r)(i_G-c_V)}{(\alpha md_r-md_Y i_r)^2}md_r t+\frac{(M_0 i_r+\gamma md_r)(i_G-c_V)}{(\alpha md_r-md_Y i_r)^2}md_r$$

(37) $$\frac{\partial Y}{\partial TR_0}=\frac{c_V md_r}{\alpha md_r-md_Y i_r}$$

(38) $$\frac{\partial Y}{\partial md_Y}=\frac{(\beta md_r-m_S i_r)}{(\alpha md_r-md_Y i_r)^2}i_r t-\frac{(M_0 i_r+\gamma md_r)}{(\alpha md_r-md_Y i_r)^2}i_r$$

(39) $$\frac{\partial Y}{\partial md_r}=\frac{m_S\alpha-\beta md_Y}{(\alpha md_r-md_Y i_r)^2}i_r t+\frac{\gamma md_Y+\alpha M_0}{(\alpha md_r-md_Y i_r)^2}i_r$$

(40) $$\frac{\partial Y}{\partial m_S}=-\frac{i_r}{\alpha md_r-md_Y i_r}t$$

(41) $$\frac{\partial Y}{\partial M_0} = -\frac{i_r}{\alpha md_r - md_Y i_r}$$

(42) $$\frac{\partial r}{\partial c_V} = \frac{\beta md_r - m_S i_r}{(\alpha md_r - md_Y i_r)^2} md_Y (1 + c_{TF} - t_Y)t + \frac{TF_0 - TR_0}{\alpha md_r - md_Y i_r} md_Y - \frac{(i_r M_0 + \gamma md_r)(1 + c_{TF} - t_Y)}{(\alpha md_r - md_Y i_r)^2} md_Y$$

(43) $$\frac{\partial r}{\partial C_0} = \frac{md_Y}{\alpha md_r - md_Y i_r}$$

(44) $$\frac{\partial r}{\partial i_G} = \frac{\beta md_r - m_S i_r}{(\alpha md_r - md_Y i_r)^2} md_Y (t_Y + i_{OR})t + \frac{md_Y (TR_0 + OR_0) + M_0 (t_Y + i_{OR})}{\alpha md_r - md_Y i_r} - \frac{(\gamma md_Y + \alpha M_0)(t_Y + i_{OR})}{(\alpha md_r - md_Y i_r)^2} md_r$$

(45) $$\frac{\partial r}{\partial i_{OR}} = \frac{\beta md_r - m_S i_r}{(\alpha md_r - md_Y i_r)^2} md_Y i_G t - \frac{i_r M_0 + \gamma md_r}{(\alpha md_r - md_Y i_r)^2} md_Y i_G$$

(46) $$\frac{\partial r}{\partial OR_0} = \frac{i_G md_Y}{\alpha md_r - md_Y i_r}$$

(47) $$\frac{\partial r}{\partial i_Y} = \frac{\beta md_r - m_S i_r}{(\alpha md_r - md_Y i_r)^2} md_Y t - \frac{M_0 i_r + \gamma md_r}{(\alpha md_r - md_Y i_r)^2} md_Y$$

(48) $$\frac{\partial r}{\partial i_r} = \frac{m_S \alpha - \beta md_Y}{(\alpha md_r - md_Y i_r)^2} md_Y t + \frac{\gamma md_Y + \alpha M_0}{(\alpha md_r - md_Y i_r)^2} md_Y$$

(49) $$\frac{\partial r}{\partial im_Y} = \frac{m_S i_r - \beta md_r}{(\alpha md_r - md_Y i_r)^2} md_Y t + \frac{M_0 i_r + \gamma md_r}{(\alpha md_r - md_Y i_r)^2} md_Y$$

(50) $$\frac{\partial r}{\partial c_{ei}} = -\frac{r_{CH} md_Y}{\alpha md_r - md_Y i_r} t - \frac{md_Y CH_0}{\alpha md_r - md_Y i_r}$$

(51) $$\frac{\partial r}{\partial IM_0} = -\frac{md_Y}{\alpha md_r - md_Y i_r}$$

(52) $$\frac{\partial r}{\partial ex_Y} = \frac{\beta md_r - m_S i_r}{(\alpha md_r - md_Y i_r)^2} md_Y t - \frac{M_0 i_r + \gamma md_r}{(\alpha md_r - md_Y i_r)^2} md_Y$$

(53) $$\frac{\partial r}{\partial c_{ee}} = \frac{r_{CH} md_Y}{\alpha md_r - md_Y i_r} t + \frac{md_Y CH_0}{\alpha md_r - md_Y i_r}$$

(54) $$\frac{\partial r}{\partial EX_0} = \frac{md_Y}{\alpha md_r - md_Y i_r}$$

(55) $$\frac{\partial r}{\partial r_{CH}}=\frac{c_{ee}-c_{ei}}{\alpha md_r-md_Y i_r}md_Y t$$

(56) $$\frac{\partial r}{\partial CH_0}=\frac{c_{ee}-c_{ei}}{\alpha md_r-md_Y i_r}md_Y$$

(57) $$\frac{\partial r}{\partial c_{TF}}=\frac{\beta md_r-m_S i_r}{(\alpha md_r-md_Y i_r)^2}c_V md_Y t-\frac{M_0 i_r+\gamma md_r}{(\alpha md_r-md_Y i_r)^2}md_Y c_V$$

(58) $$\frac{\partial r}{\partial TF_0}=\frac{c_V md_Y}{\alpha md_r-md_Y i_r}$$

(59) $$\frac{\partial r}{\partial t_Y}=\frac{\beta md_r-m_S i_r}{(\alpha md_r-md_Y i_r)^2}(i_G-c_V)md_Y t-\frac{M_0 i_r+\gamma md_r}{(\alpha md_r-md_Y i_r)^2}(i_G-c_V)md_Y$$

(60) $$\frac{\partial r}{\partial TR_0}=-\frac{c_V md_Y}{\alpha md_r-md_Y i_r}$$

(61) $$\frac{\partial r}{\partial md_Y}=\frac{m_S i_r-\beta md_r}{(\alpha md_r-md_Y i_r)^2}\alpha t+\frac{\gamma md_r+M_0 i_r}{(\alpha md_r-md_Y i_r)^2}\alpha$$

(62) $$\frac{\partial r}{\partial md_r}=\frac{\beta md_Y-m_S\alpha}{(\alpha md_r-md_Y i_r)^2}\alpha t-\frac{\gamma md_Y+\alpha M_0}{(\alpha md_r-md_Y i_r)^2}\alpha$$

(63) $$\frac{\partial r}{\partial m_S}=\frac{\alpha}{\alpha md_r-md_Y i_r}t$$

(64) $$\frac{\partial r}{\partial M_0}=\frac{\alpha}{\alpha md_r-md_Y i_r}$$

5 The variations of equilibrium output and real interest rate based on the parameter values

In what follows let consider the dynamic equations of the model:

(65) $$\begin{cases}\dfrac{dY}{dt}=A(D(t)-Y(t))\\ \dfrac{dr}{dt}=B(MD(t)-MS(t))\end{cases},\ A,B\in\mathbf{R}$$

With notations (26) we have:

(66) $D(t)=(\alpha+1)Y(t)+i_r r(t)-\beta t+\gamma$

(67) $MD(t)=md_Y Y(t)+md_r r(t)$

The system becomes:

$$(68)\quad \begin{cases} \dfrac{dY}{dt} = A\alpha Y(t) + Ai_r r(t) - A\beta t + A\gamma \\ \dfrac{dr}{dt} = Bmd_Y Y(t) + Bmd_r r(t) - Bm_S t - BM_0 \end{cases}$$

or, in matrix notation:

$$(69)\quad \begin{pmatrix} \dfrac{dY}{dt} \\ \dfrac{dr}{dt} \end{pmatrix} = \begin{pmatrix} A\alpha & Ai_r \\ Bmd_Y & Bmd_r \end{pmatrix} \begin{pmatrix} Y(t) \\ r(t) \end{pmatrix} + \begin{pmatrix} -A\beta t + A\gamma \\ -Bm_S t - BM_0 \end{pmatrix}$$

and it is a system of differential equations of first order, linear, with constant coefficients satisfying the initial conditions: Y(year_1)=Y_0, r(year_1)=r_0 where year_1 is the first year of analysis.

Let now the matrix of the system:

$$(70)\quad M = \begin{pmatrix} A\alpha & Ai_r \\ Bmd_Y & Bmd_r \end{pmatrix}$$

and the characteristic equation for eigenvalues determination:

$$(71)\quad \begin{vmatrix} A\alpha - \lambda & Ai_r \\ Bmd_Y & Bmd_r - \lambda \end{vmatrix} = 0$$

that is: $\lambda^2 - (A\alpha + Bmd_r)\lambda + AB(\alpha md_r - i_r md_Y) = 0$. Let the discriminant of the equation:

$$(72)\quad \Delta = (A\alpha - Bmd_r)^2 + 4ABi_r md_Y$$

and $\lambda_1, \lambda_2 \in \mathbf{C}$ the eigenvalues.

For the beginning we must determine a particular solution of the system (94).

<u>Case p.1</u> $\alpha md_r - i_r md_Y \neq 0$

In this case a particular solution has the expression: $\begin{cases} Y_0(t) = at + b \\ r_0(t) = ct + d \end{cases}$ where a,b,c,d$\in\mathbf{R}$ are determined after replacing in the system (93).

<u>Case p.2</u> $\alpha md_r - i_r md_Y = 0,\ A\alpha + Bmd_r \neq 0$

In this case a particular solution has the expression: $\begin{cases} Y_0(t) = t(at + b) \\ r_0(t) = t(ct + d) \end{cases}$ where a,b,c,d$\in\mathbf{R}$ are determined after replacing in the system (93).

<u>Case p.3</u> $\alpha md_r - i_r md_Y = 0,\ A\alpha + Bmd_r = 0$

In this case a particular solution has the expression: $\begin{cases} Y_0(t) = t^2(at+b) \\ r_0(t) = t^2(ct+d) \end{cases}$ where a,b,c,d∈**R** are determined after replacing in the system (93).

After the determination of particular solution, we have the following cases for the solution of homogenous system: $\begin{pmatrix} \frac{dY}{dt} \\ \frac{dr}{dt} \end{pmatrix} = \begin{pmatrix} A\alpha & Ai_r \\ Bmd_Y & Bmd_r \end{pmatrix} \begin{pmatrix} Y(t) \\ r(t) \end{pmatrix}$.

Case o.1 $\Delta>0$ ($\lambda_1 \neq \lambda_2$)

The solution is: $\begin{cases} Y_{hom}(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \\ r_{hom}(t) = C_3 e^{\lambda_1 t} + C_4 e^{\lambda_2 t} \end{cases}$

where $C_1,C_2,C_3,C_4 \in$ **R** will be determined by replacing in the homogenous system.

Case o.2 $\Delta=0$ ($\lambda_1=\lambda_2=\lambda$)

The solution is: $\begin{cases} Y_{hom}(t) = (C_1 t + C_2)e^{\lambda t} \\ r_{hom}(t) = (C_3 t + C_4)e^{\lambda t} \end{cases}$

where $C_1,C_2,C_3,C_4 \in$ **R** will be determined by replacing in the homogenous system.

Case o.3 $\Delta<0$ ($\lambda_1=\alpha_1+i\beta_1$, $\lambda_2=\alpha_1-i\beta_1$)

The solution is: $\begin{cases} Y_{hom}(t) = C_1 e^{\alpha_1 t}\cos\beta_1 t + C_2 e^{\alpha_1 t}\sin\beta_1 t \\ r_{hom}(t) = C_3 e^{\alpha_1 t}\cos\beta_1 t + C_4 e^{\alpha_1 t}\sin\beta_1 t \end{cases}$

where $C_1,C_2,C_3,C_4 \in$ **R** will be determined by replacing in the homogenous system.

Finally the general solution will be:

$$(73)\quad \begin{cases} Y(t) = Y_{hom}(t) + Y_0(t) \\ r(t) = r_{hom}(t) + r_0(t) \end{cases}$$

which is dependent on two arbitrary constants. From the initial conditions: Y(year_1)=Y_0, r(year_1)=r_0 there will be determined.

6 Application of the model to the Romanian economy

After the regression analysis we find:

(74) c_V=1.062338107, C_0=-21306.522399, i_G=0.281763291, i_{OR}=0.077131491, OR_0=8586.917756, r_{CH}=0.112581319, CH_0=-222.1708473, c_{TF}=0.353272369, TF_0=-24079.51702, t_Y=0.395122134, TR_0=-25436.01202, m_S=2745.9441, M_0=-5471920.509, i_Y=0.308842141,
i_r=-1301.197683, im_Y=2.468228803, c_{ei}=-20686.68561, IM_0=-117531.7752, ex_Y=0.970442258, c_{ee}=606.9387431, EX_0=-57581.34747, md_Y=0.416549399, md_r=-2860.243226,
α=-1.03800116, β=-2397.26431, γ=-4695485.06

Computing now for the equilibrium the values of GDP and of Real Interest Rate for each year from the period and after replacing in (28)-(40) we find the following situations.

Table 1

Year	GDP real - Y	GDP for equilibrium - Y*
2001	85820.2	91058.11
2002	90269.27724	94028.75
2003	95255.99981	96999.39
2004	103218.094	99970.04
2005	107524.2492	102940.7
2006	116185.9298	105911.3
2007	124160.639	108882
2008	134663.3768	111852.6
2009	125146.9341	114823.2
2010	124147.6909	117793.9
2011	125459.0429	120764.5
2012	126263.2192	123735.2
2013	130722.3328	126705.8
2014	134590.4634	129676.5

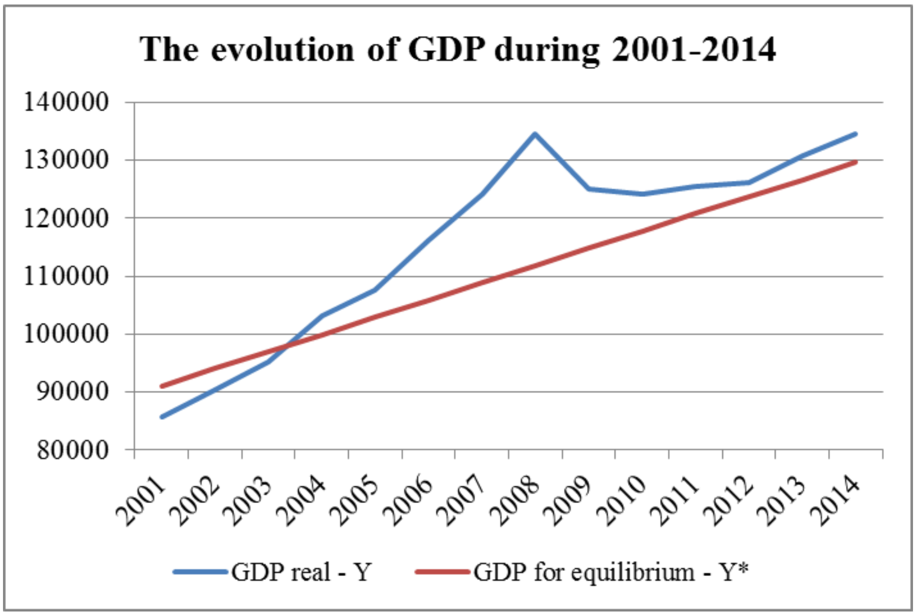

Figure 1

The analysis of GDP growth in the analyzed period reflects different situations. Thus, during 2001-2003, real GDP level was below the equilibrium, which somehow justified by the relocation of economy to one capitalist after the complicated decade at the end of the century.

The second period 2004-2009, especially after 2005, was under the influence of liberal policies to stimulate consumption which led to a disproportionate rise in GDP far above the real possibilities of the Romanian economy. As we will see below, consumption growth was made, in particular based on massive imports, a lending to households with no sense and safety rule. Enlightening this is 2008 when the world economy into recession started and consumption in Romania reached paroxysmal.

After 2009, the real GDP starts to approach the balance, although still high, beaing clearly influenced by the strong economic crisis that affected Romania.

Table 2

Year	Actual final consumption of households real - C	Actual final consumption of households for equilibrium - C*
2001	67758.83	72820.71
2002	70876.21	75844.47
2003	74269.63	78868.23
2004	83028.18	81891.98
2005	92658.84	84915.74
2006	103566.1	87939.5
2007	137896.7	90963.25
2008	156482.3	93987.01
2009	107423	97010.77
2010	109358.3	100034.5
2011	116227.7	103058.3
2012	121122.3	106082
2013	112366.5	109105.8
2014	117094.6	112129.5

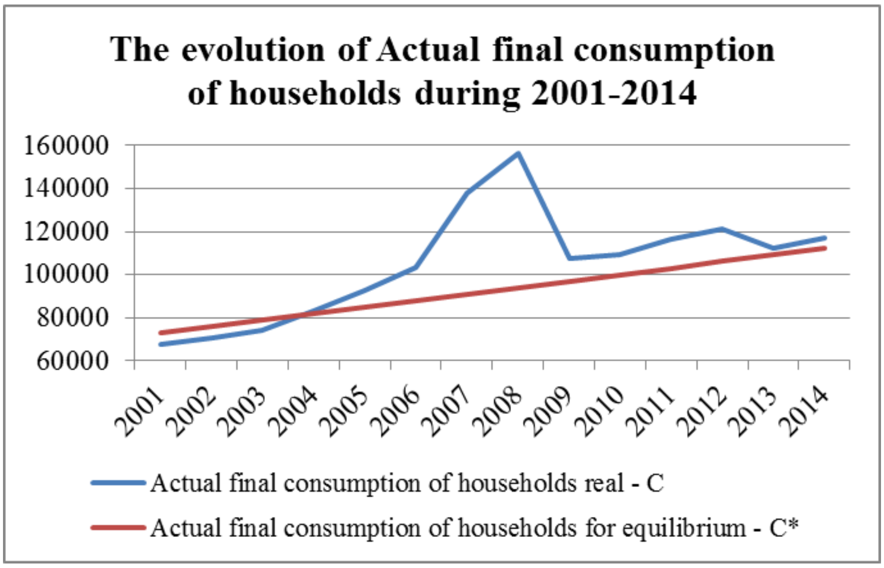

Figure 2

Table 3

Year	Disposable Income real - DI	Disposable Income for equilibrium - DI*
2001	84098.61	88603.84
2002	88546.04	91450.16
2003	92573.44	94296.49
2004	97305.46	97142.81
2005	106691.5	99989.13
2006	113882.8	102835.5
2007	148041.2	105681.8
2008	170114.5	108528.1
2009	121612.2	111374.4
2010	123329.5	114220.7
2011	127768.7	117067.1
2012	130399.3	119913.4
2013	127005.3	122759.7
2014	133280.9	125606

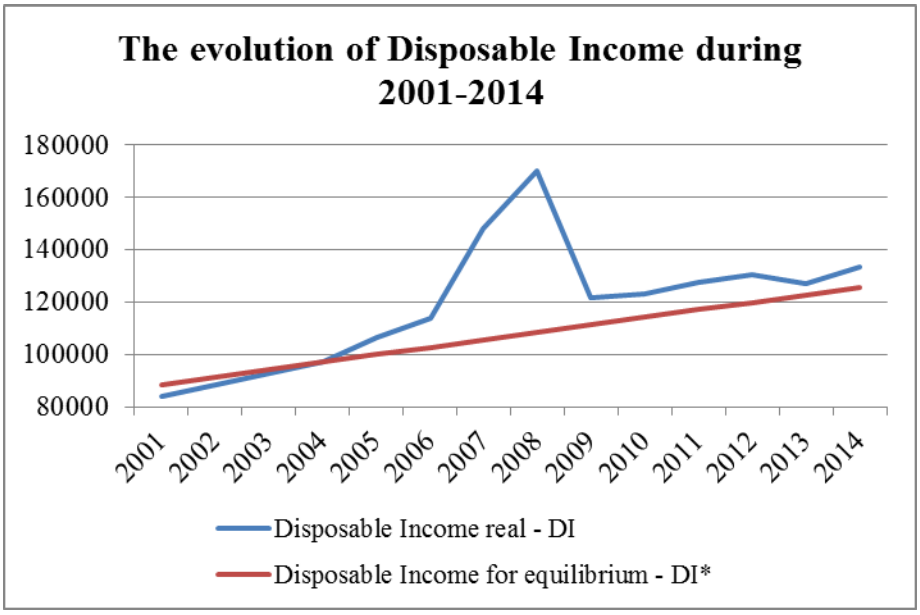

Figure 3

Analysis of household consumption and disposable income reflects an apparently paradoxical. First, one should note the marginal propensity to consume c_V=1.06 whose value (indeed, statistically determined relative to the entire period) exceeds theoretical considerations, normally at odds subunit. Although the regression equation for 2010-2014 recalculated marginal propensity to consume to 0.967 it remains extraordinarily high.

Comparative analysis of the evolution of Disposable Income and Actual final consumption of households during 2001-2014 reflects a share of consumption in Disposable Income between 80% (2002-2003) and over 93% (2007) (figure 4). If until the year 2003 the situation can be understood against the background of an adaptation of the consumption needs of the modern world, after this year it is again symptomatic of the mess Romanian economy. In 2007, the share of 93.15% in Disposable Income related to consumer credit expansion to households shows an endowment hysteria especially consumer goods far beyond the common man. The emergence of the economic crisis has tempered the phenomenon to a very small extent, even if Disposable Income decreased, consumption share remained very high, hovering somewhere at 88-93%. This reflects low economic education of the Romanian population, justified with distrust in the future, due to possible inflation that actual savings may decrease.

Analysis of changes in consumption during 2007-2014, deposits and loans, even if statistical provides a very low correlation between them, reveals an increase of 13.48% in 2008 to a growth of consumption loans of the population – 28.53%. Also, a paradoxical situation was in 2011 when credit was reduced from the previous year to 3.09%, disposable income increased by 3.60%, but consumption was increased by 6.28% while only declarative Romania was out of the crisis. Again in 2013, the disposable income fell by 2.60% and consumption by 7.23%. This situation can be explained easily by decreasing 4.25% crediting. As a conclusion, it emerges that the evolution of consumption is dependent simultaneously from the change in disposable income and household lending.

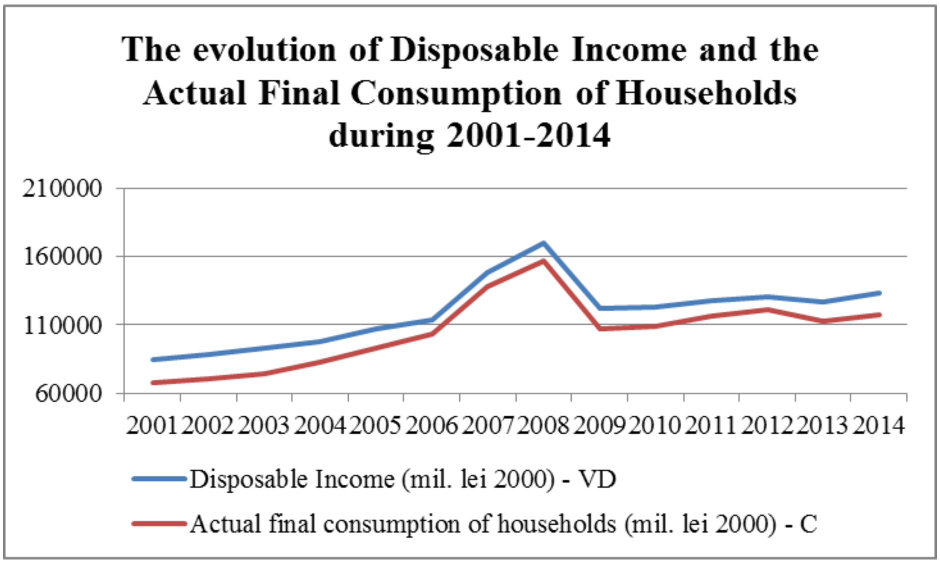

Figure 4

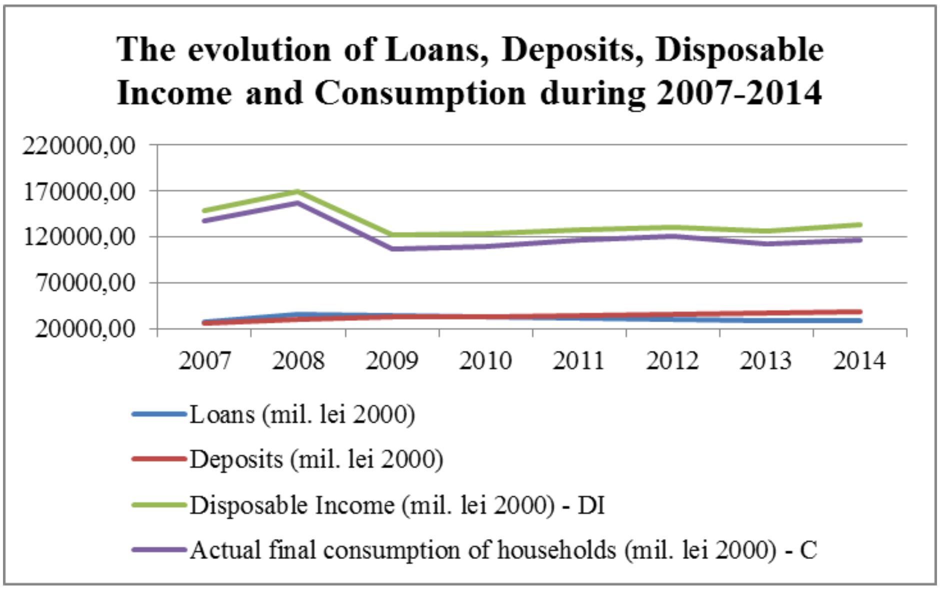

Figure 5

Table 4

Year	The actual final consumption of the government real - G	The actual final consumption of the government for equilibrium - G*
2001	6288.275	7369.076
2002	6198.451	7764.36
2003	9655.797	8159.645
2004	8478.723	8554.93
2005	10846.86	8950.215
2006	10089.7	9345.5
2007	13961.99	9740.785
2008	16345.71	10136.07
2009	11696.16	10531.35
2010	10681.44	10926.64
2011	10181.21	11321.92
2012	10559.68	11717.21
2013	11511.67	12112.49
2014	12897.15	12507.78

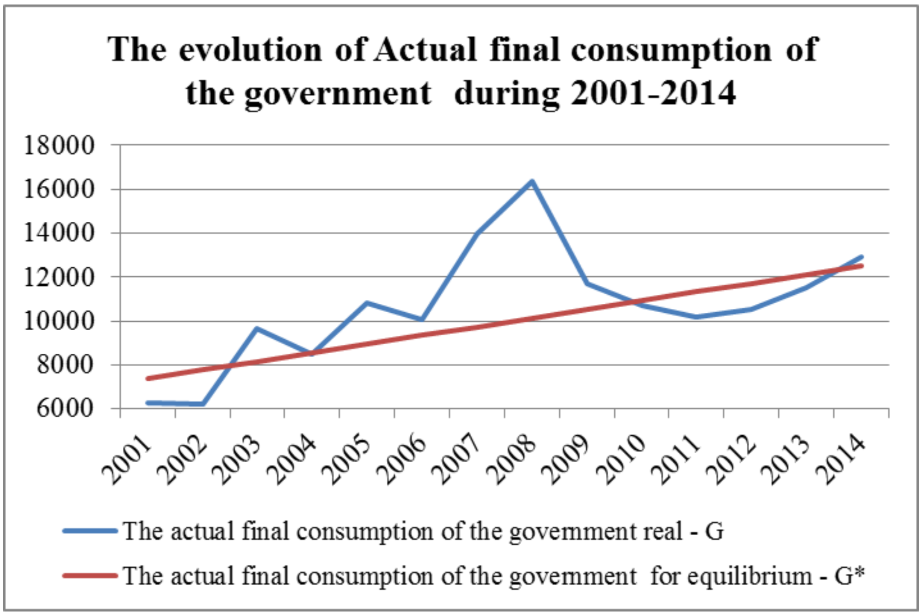

Figure 6

Analysis of government consumption proves again irresponsibility in public spending. Thus, in 2007-2009 they amounted to enormous value to the equilibrium level even in the last part of the period when there were obvious signs of the economic crisis. With the austerity measures taken in 2010, they fell far below the equilibrium level which meant a restructuring of the bureaucracy, but not enough.

Table 5

Year	Real Interest Rate - r (%)	Real Interest Rate for equilibrium - r* (%)
2001	3.2	5.320025
2002	4.87	4.792614
2003	3.44	4.265202
2004	7.09	3.737791
2005	1.99	3.21038
2006	1.84	2.682968
2007	2.4	2.155557
2008	1.71	1.628146
2009	3.28	1.100734
2010	0.34	0.573323
2011	0.39	0.045912
2012	1.85	-0.4815
2013	1.56	-1.00891
2014	1.88	-1.53632

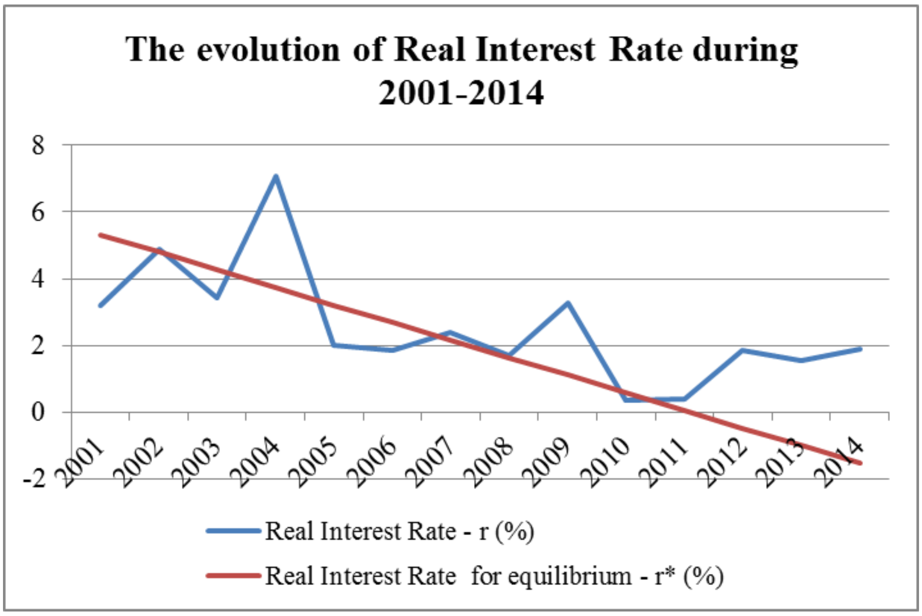

Figure 7

Relative to the real interest rate in the period analyzed it oscillated around balance. But here are a series of interesting issues. In 2004 it was located at a value more than 3 percentage points higher than the optimal (in terms of mathematical model), which however has not been seen in the fall in investments, which are very close to the equilibrium level. The next period, 2005-2006 represented a decrease of approximately 1.5 percentage points below the equilibrium rate which was reflected in credit growth with negative effects on consumption that we have analyzed above. In 2010-2011, the real rate ranged around equilibrium which contributed to an increase in real investment. Since 2012, the real rate was again over the balance, having suffered again investments especially in the post-crisis period that would have to generate a new impetus to the Romanian economy.

Table 6

Year	Investments real - I	Investments for equilibrium - I*
2001	18663.1	21200.18
2002	19493.34	22803.9
2003	21214.23	24407.63
2004	26643.54	26011.35
2005	25779.78	27615.08
2006	35233.9	29218.81
2007	36134.58	30822.53
2008	32262.35	32426.26
2009	39412.95	34029.98
2010	40779.26	35633.71
2011	40235.91	37237.44
2012	34310.17	38841.16
2013	40352.81	40444.89
2014	42033.73	42048.62

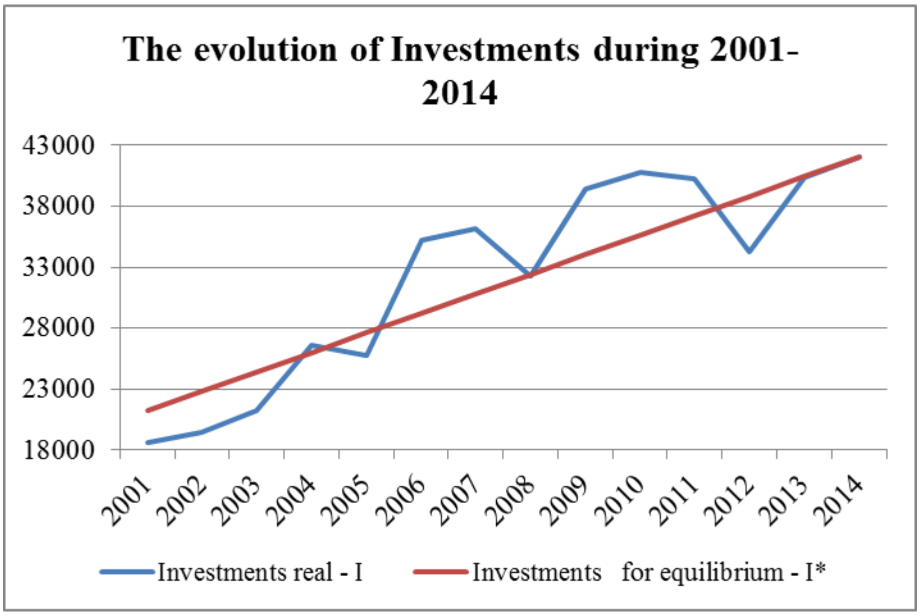

Figure 8

Increasing investment would seem, at first glance, favorable, hovering mostly above the equilibrium level. From Figure 8 it can be seen easily that social-democratic regimes of the periods 2001-2004, 2012-2014 respectively, placed investments suboptimal what was seen, especially in the first period, to the suboptimal situation of GDP. In the period 2006-2011 investments known, at least in value terms, a very large scale, but the problem is that about their quality and direction and not about their volume. Investments in infrastructure which claimed huge costs without finality, referring to roads, or capacities that subsequently were exploited , example of hospitals constructed, equipped and then dismantled, prove an investment activity without clearly outlined direction.

Table 7

Year	Exports real - EX	Exports for equilibrium - EX*
2001	29755.6	32669.45
2002	34785.2	35620.62
2003	38135.36	38571.78
2004	42965.73	41522.95
2005	48012.61	44474.12
2006	53281.08	47425.29
2007	68858.56	50376.45
2008	70901.38	53327.62
2009	49910.08	56278.79
2010	58177.35	59229.96
2011	68029.9	62181.12
2012	70800.84	65132.29
2013	79950.3	68083.46
2014	88336.05	71034.62

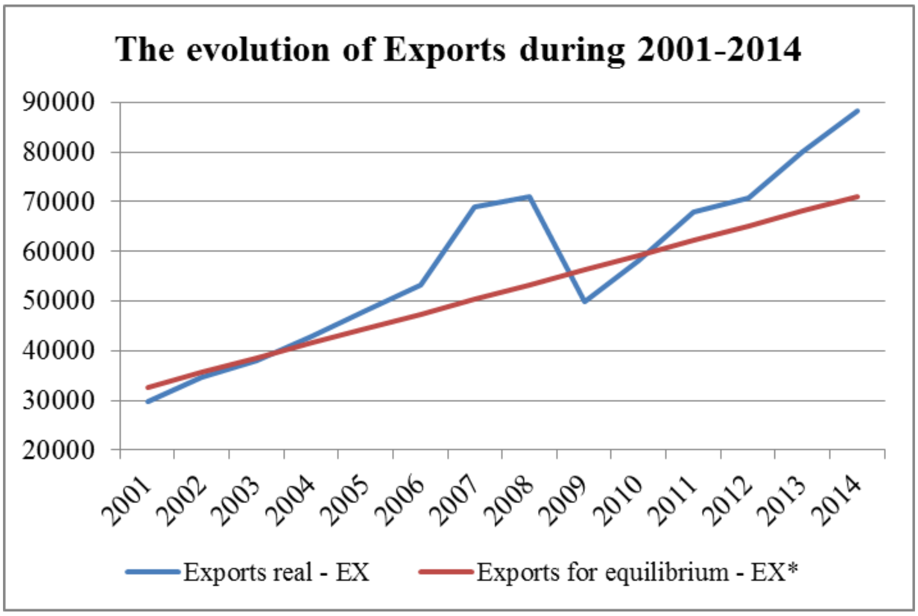

Figure 9

Table 8

Year	Imports real - IM	Imports for equilibrium - IM*
2001	36645.6	43001.31
2002	41083.92	48004.6
2003	48019.02	53007.89
2004	57898.08	58011.19
2005	69773.84	63014.48
2006	85984.83	68017.77
2007	132691.2	73021.06
2008	141328.4	78024.35
2009	83295.29	83027.64
2010	94848.69	88030.94
2011	109215.6	93034.23
2012	110529.8	98037.52
2013	113459	103040.8
2014	125771.1	108044.1

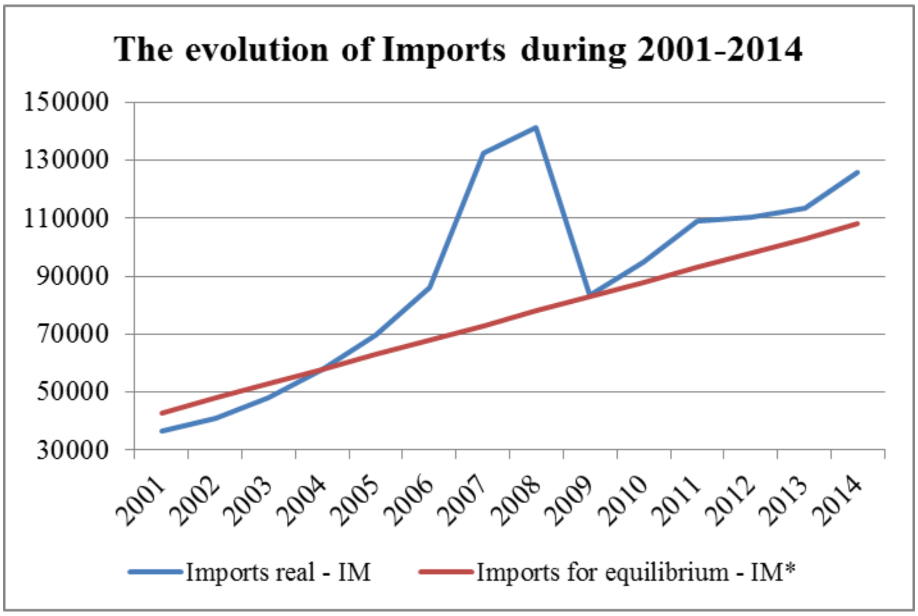

Figure 10

Table 9

Year	Net Exports real - NX	Net Exports for equilibrium - NX*
2001	-6890	-10331.9
2002	-6298.72	-12384
2003	-9883.66	-14436.1
2004	-14932.3	-16488.2
2005	-21761.2	-18540.4
2006	-32703.7	-20592.5
2007	-63832.7	-22644.6
2008	-70427	-24696.7
2009	-33385.2	-26748.9
2010	-36671.3	-28801
2011	-41185.7	-30853.1
2012	-39728.9	-32905.2
2013	-33508.7	-34957.4
2014	-37435	-37009.5

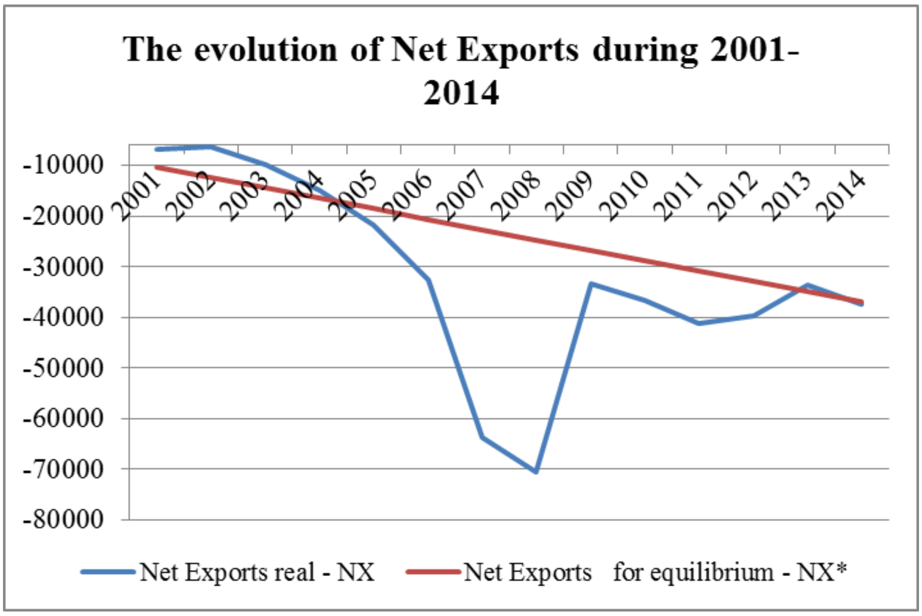

Figure 11

Romania's foreign trade evolution best explains the need for radical restructuring of the national economy. From Figure 11 we can see that if, in real terms, in 2001-2004 even though the trade balance was negative, overall net exports was above the equilibrium, in the next period, the so-called boom has been a disaster . Stimulating consumption was done almost exclusively on imports, which were in 2007-2008 almost two times higher than the equilibrium level. Lack of domestic production capacities, referring specifically to consumer durables, consumer loans with bulletin led to a massive demand from people for import products which led to a huge imbalance in the trade balance. The beginning of the economic crisis tempered enthusiasm and the gap was reduced. Forecast model of balance is negative, however, the current state of the Romanian economy by emphasizing leading trade deficit.

Table 10

Year	Total Income real - TI	Total Income for equilibrium - TI*
2001	25511.03	26153.43
2002	26506.91	27556.32
2003	28050.69	28959.22
2004	30400.21	30362.12
2005	33335.19	31765.01
2006	36957.62	33167.91
2007	46694.96	34570.81
2008	55937.3	35973.7
2009	37707.38	37376.6
2010	38814.42	38779.5
2011	41771.27	40182.39
2012	43657.57	41585.29
2013	41321.81	42988.19
2014	44115.83	44391.09

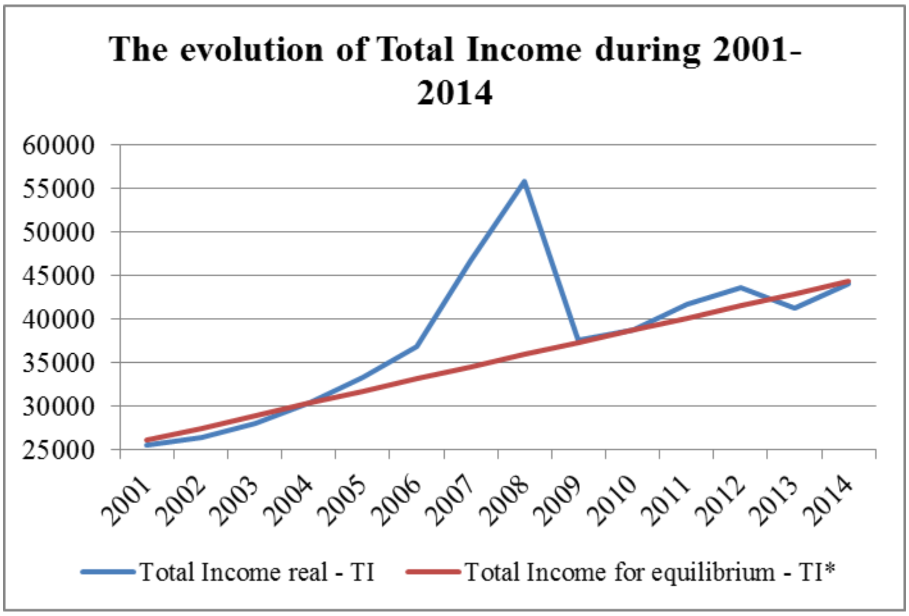

Figure 12

Total Income reveals a situation theoretically favorable, mostly they are above the balance. What should be noted is the fact that statistical analysis takes into account existing revenues and not those who could come through reducing tax evasion. However, it may be noted that in 2005-2008, due to lower tax rate to 16%, the actual level of Total Income has greatly increased which contributed essentially to alleviate imbalances State Budget.

Table 11

Year	Tax rate real - TR	Tax rate for equilibrium - TR*
2001	10565.3	10543.06
2002	10604.57	11716.83
2003	12205.86	12890.59
2004	13314.69	14064.36
2005	14054.55	15238.13
2006	22057.47	16411.9
2007	28054.03	17585.66
2008	32048.22	18759.43
2009	20882.52	19933.2
2010	21399.06	21106.96
2011	24037.53	22280.73
2012	25777.67	23454.5
2013	24563.21	24628.26
2014	25783.26	25802.03

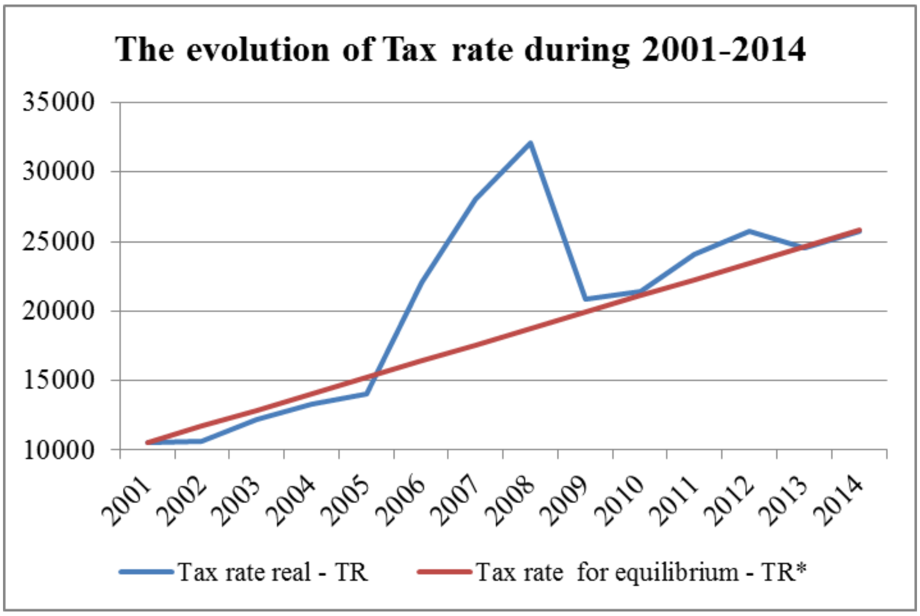

Figure 13

Tax Rate evolution follows essentially the same trend with the Total Income, largely above the equilibrium level, in 2005-2008, due to reduced tax rate to 16%, the greatly increasing of actual Tax Rate being 56.94% - from 2005 to 2006 and 27.19% - 2007 versus 2006. The last years of analysis (2013-2014) again shows a sinuous evolution of this indicator, after 2013 when there was a decrease of 4.71% (on the background of legislative changes and introduction of additional taxes), in 2014 returning to a growth of 4.97 %.

Table 12

Year	Fiscality rate real TR/Y (%)	Fiscality rate for equilibrium - TR*/Y* (%)
2001	12.31%	11.58%
2002	11.77%	12.46%
2003	12.79%	13.29%
2004	13.04%	14.07%
2005	12.72%	14.80%
2006	18.38%	15.50%
2007	18.26%	16.15%
2008	18.09%	16.77%
2009	17.06%	17.36%
2010	17.42%	17.92%
2011	18.53%	18.45%
2012	19.16%	18.96%
2013	18.69%	19.44%
2014	18.72%	19.90%

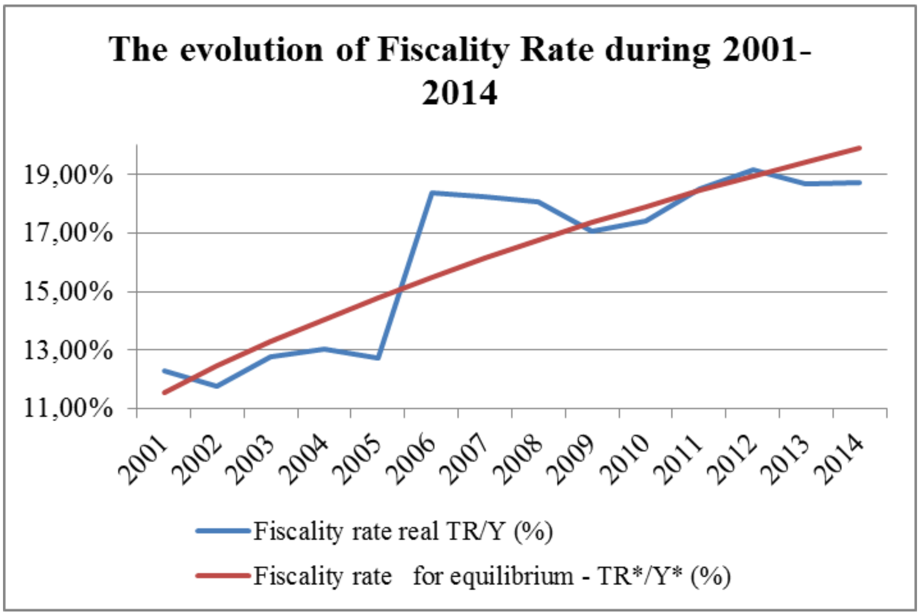

Figure 14

What should be noted is the evolution of Fiscality Rate (the ratio between Tax Rate and GDP). If during the social-democratic regime or those of transition (2002-2005, 2013-2014) it was below the balance, primarily due to overly high taxes that led to modest revenue, during the 2006-2008 fiscal development was a favorable one, leading to higher receipts to the State Budget, primarily due to flat tax level of 16%.

Table 13

Year	Other revenues real - OR	Other revenues for equilibrium - OR*
2001	14945.72	15610.37
2002	15902.34	15839.5
2003	15844.83	16068.63
2004	17085.52	16297.76
2005	19280.64	16526.89
2006	14900.15	16756.02
2007	18640.92	16985.15
2008	23889.08	17214.28
2009	16824.86	17443.41
2010	17415.36	17672.54
2011	17733.74	17901.67
2012	17879.9	18130.8
2013	16758.59	18359.93
2014	18332.57	18589.06

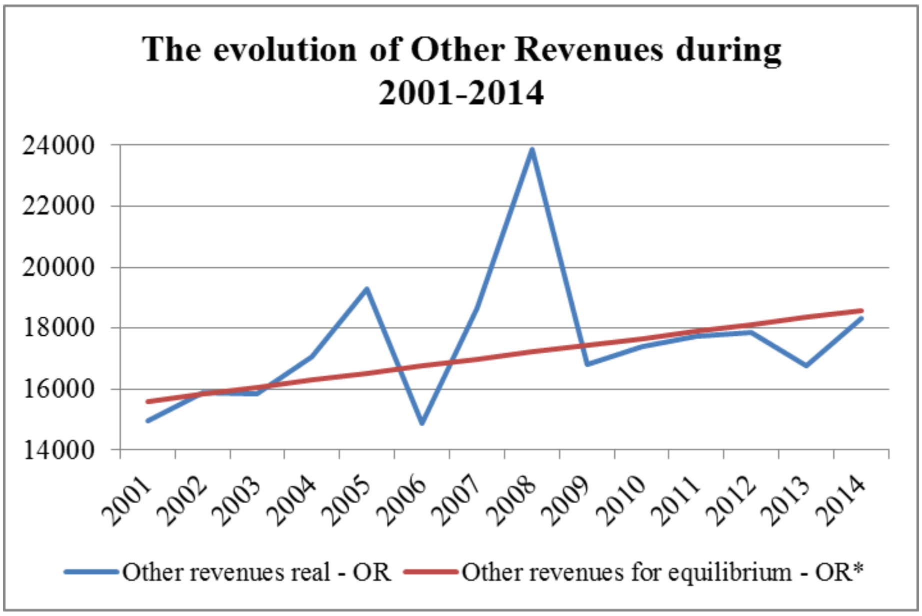

Figure 15

The evolution of Other Revenues indicator experienced a fluctuating trend which has a character more or less short term.

Table 14

Year	Government transfers real - TF	Government transfers for equilibrium - TF*
2001	8843.708	8088.796
2002	9026.951	9138.242
2003	9371.477	10187.69
2004	8492.239	11237.13
2005	10240.93	12286.58
2006	15956.71	13336.03
2007	22442.4	14385.47
2008	24974.67	15434.92
2009	20070.41	16484.36
2010	21874.64	17533.81
2011	22052.25	18583.26
2012	21605.22	19632.7
2013	20110.36	20682.15
2014	21336.87	21731.59

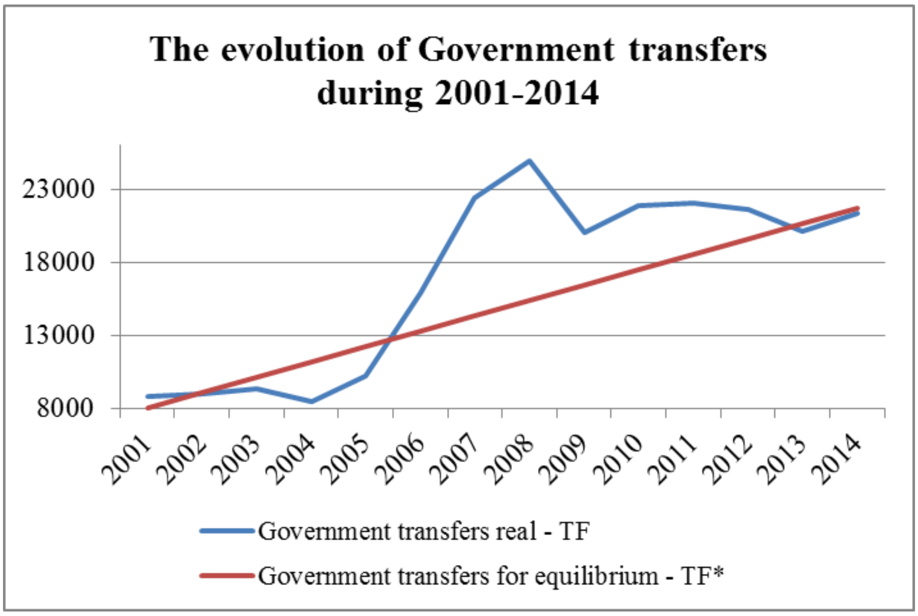

Figure 16

The Government Transfers were generally located above the balance. Social policies in Romania have been known for having sinuous developments in political regimes that have succeeded at intervals of 4 years. Thus, if in the 2002-2005 period they were below equilibrium, in the next they experienced exaggerated levels reaching in 2008 more than 160% of the optimal level, leading to major imbalances. Since 2013 they are about on optimal line which gives hopes to rebalance the State Budget.

Table 15

Year	Exchange rate real - CH	Exchange rate for equilibrium - CH*
2001	2.6012	3.104371
2002	3.1241	3.216953
2003	3.7559	3.329534
2004	4.0523	3.442115
2005	3.6234	3.554697
2006	3.5245	3.667278
2007	3.3373	3.779859
2008	3.6827	3.892441
2009	4.2373	4.005022
2010	4.2099	4.117603
2011	4.2379	4.230185
2012	4.456	4.342766
2013	4.419	4.455347
2014	4.4446	4.567929

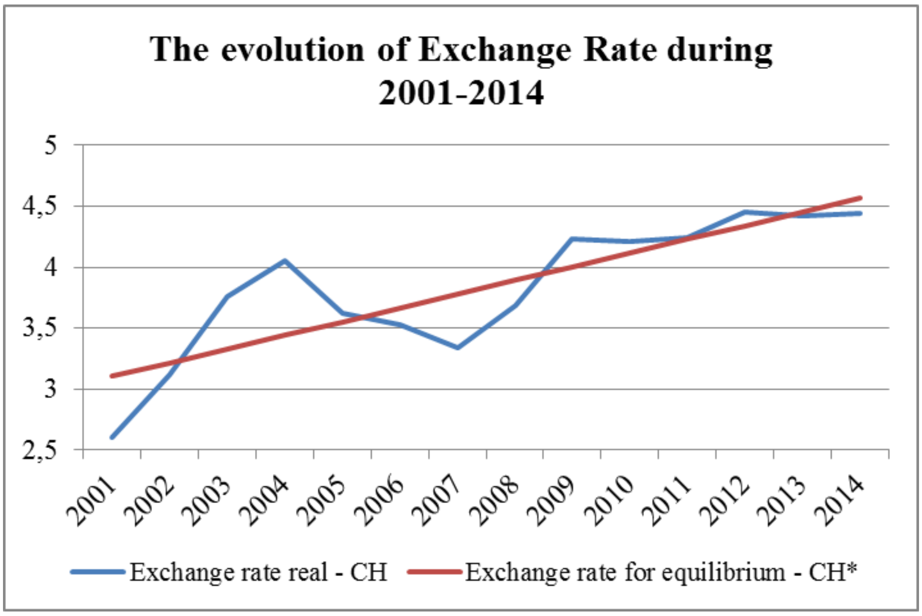

Figure 17

The exchange rate, which at first had a sinuous evolution (being extremely high in 2003-2004 and then in 2006-2007 far below the equilibrium level - that favored massive imports) recorded after Romania's EU integration a level usually located very close to the equilibrium, which proves a fair policy for determining it.

Table 16

Year	**Money Demand real - MD**	**Money Demand for equilibrium - MD***
2001	19619.66	20531.79
2002	22066.63	23893.57
2003	22116.15	27479.31
2004	26465.94	32304.44
2005	32802.96	35606.69
2006	38187.59	40723.24
2007	54385.87	45553.62
2008	58668.23	51437.06
2009	45085.86	48981.51
2010	45924.48	50073.8
2011	48691.6	52128.57
2012	50140.47	53972.07
2013	49751.52	57338.04
2014	53964.85	60457.83

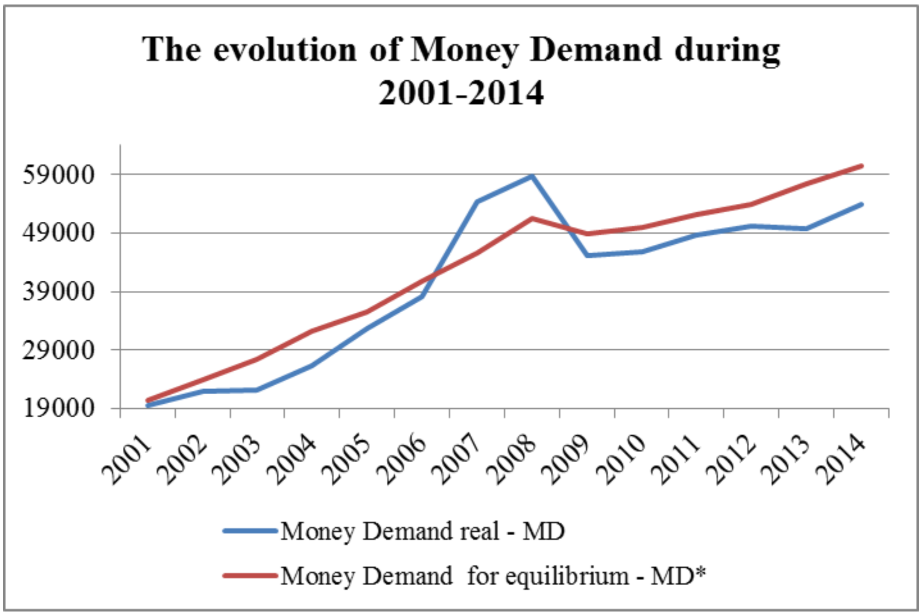

Figure 18

Finally, the Money Demand was located almost consistently below the equilibrium level except for the period 2007-2008 when has been far above because, first, high liquidity compared with other periods.

The second part of the analysis concerns the sensitivity of the two basic indicators: GDP and the real interest rate depending on the model parameters. From formulas (44)-(89) with parameter values in (99), we get:

(75) $\frac{\partial Y}{\partial c_V}=-4567728.0137+2318.7958t$, $\frac{\partial Y}{\partial C_0}=0.8147$,

$\frac{\partial Y}{\partial i_G}=-2265615.3687+1142.8894t$

(76) $\frac{\partial Y}{\partial i_{OR}}=-1343556.9951+681.8884t$, $\frac{\partial Y}{\partial OR_0}=0.2295$,

$\frac{\partial Y}{\partial i_Y}=-4768389.0629+2420.0754t$

(77) $\frac{\partial Y}{\partial i_r}=864.0896-0.4297t$, $\frac{\partial Y}{\partial im_Y}=4768389.0629-2420.0754t$,

$\frac{\partial Y}{\partial c_{ei}}=180.9946-0.0917t$

(78) $\frac{\partial Y}{\partial IM_0}=-0.8147$, $\frac{\partial Y}{\partial ex_Y}=-4768389.0629+2420.0754t$,

$$\frac{\partial Y}{\partial c_{ee}} = -180.9946 + 0.0917t$$

(79) $\frac{\partial Y}{\partial EX_0} = 0.8147$, $\frac{\partial Y}{\partial r_{CH}} = 17347.1465t$, $\frac{\partial Y}{\partial CH_0} = 17347.1465$,

$$\frac{\partial Y}{\partial c_{TF}} = -5065641.4105 + 2570.9383t$$

(80) $\frac{\partial Y}{\partial TF_0} = 0.8654$, $\frac{\partial Y}{\partial t_Y} = 3722084.4154 - 1889.0499t$, $\frac{\partial Y}{\partial TR_0} = -0.6359$

(81) $\frac{\partial Y}{\partial md_Y} = 2169261.9508 - 1100.9541t$, $\frac{\partial Y}{\partial md_r} = -393.0964 + 0.1955t$, $\frac{\partial Y}{\partial m_S} = 0.3706t$

(82) $\frac{\partial Y}{\partial M_0} = 0.3706$

(83) $\frac{\partial r}{\partial c_V} = -665.2177 + 0.3377t$, $\frac{\partial r}{\partial C_0} = 0.0001186$, $\frac{\partial r}{\partial i_G} = -329.9512 + 0.1664t$

(84) $\frac{\partial r}{\partial i_{OR}} = -195.6679 + 0.0993t$, $\frac{\partial r}{\partial OR_0} = 0.00003343$, $\frac{\partial r}{\partial i_Y} = -694.4408 + 0.3524t$

(85) $\frac{\partial r}{\partial i_r} = 0.1258 - 0.00006257t$, $\frac{\partial r}{\partial im_Y} = 694.4408 - 0.3524t$,

$$\frac{\partial r}{\partial c_{ei}} = 0.02636 - 0.00001336t$$

(86) $\frac{\partial r}{\partial IM_0} = -0.0001186$, $\frac{\partial r}{\partial ex_Y} = -694.4408 + 0.3524t$, $\frac{\partial r}{\partial c_{ee}} = -0.02636 + 0.00001336t$

(87) $\frac{\partial r}{\partial EX_0} = 0.0001186$, $\frac{\partial r}{\partial r_{CH}} = 2.5263t$, $\frac{\partial r}{\partial CH_0} = 2.5263$, $\frac{\partial r}{\partial c_{TF}} = -737.7309 + 0.3744t$

(88) $\frac{\partial r}{\partial TF_0} = 0.0001260$, $\frac{\partial r}{\partial t_Y} = 542.0630 - 0.2751t$, $\frac{\partial r}{\partial TR_0} = -0.00009261$,

(89) $\frac{\partial r}{\partial md_Y} = -1730.4799 + 0.8783t$, $\frac{\partial r}{\partial md_r} = 0.3136 - 0.0001559t$, $\frac{\partial r}{\partial m_S} = -0.0002956t$

(90) $\frac{\partial r}{\partial M_0} = -0.0002956$

For the last year of analysis – 2014 we obtain:

(91) $\frac{\partial Y}{\partial c_V}=102326.7017$, $\frac{\partial Y}{\partial C_0}=0.8147$, $\frac{\partial Y}{\partial i_G}=36163.8176$, $\frac{\partial Y}{\partial i_{OR}}=29766.2455$, $\frac{\partial Y}{\partial OR_0}=0.2295$

(92) $\frac{\partial Y}{\partial i_Y}=105642.7380$, $\frac{\partial Y}{\partial i_r}=-1.2516$, $\frac{\partial Y}{\partial im_Y}=-105642.7380$, $\frac{\partial Y}{\partial c_{ei}}=-3.7213$, $\frac{\partial Y}{\partial IM_0}=-0.8147$

(93) $\frac{\partial Y}{\partial ex_Y}=105642.7380$, $\frac{\partial Y}{\partial c_{ee}}=3.7213$, $\frac{\partial Y}{\partial EX_0}=0.8147$, $\frac{\partial Y}{\partial r_{CH}}=34937152.9507$,

(94) $\frac{\partial Y}{\partial CH_0}=17347.1465$, $\frac{\partial Y}{\partial c_{TF}}=112228.3063$, $\frac{\partial Y}{\partial TF_0}=0.8654$, $\frac{\partial Y}{\partial t_Y}=-82462.0608$

(95) $\frac{\partial Y}{\partial TR_0}=-0.6359$, $\frac{\partial Y}{\partial md_Y}=-48059.5792$, $\frac{\partial Y}{\partial md_r}=0.5694$, $\frac{\partial Y}{\partial m_S}=746.4114$, $\frac{\partial Y}{\partial M_0}=0.3706$

(96) $\frac{\partial r}{\partial c_V}=14.9023$, $\frac{\partial r}{\partial C_0}=0.0001186$, $\frac{\partial r}{\partial i_G}=5.2667$, $\frac{\partial r}{\partial i_{OR}}=4.3350$,

$\frac{\partial r}{\partial OR_0}=0.00003343$

(97) $\frac{\partial r}{\partial i_Y}=15.3852$, $\frac{\partial r}{\partial i_r}=-0.0001823$, $\frac{\partial r}{\partial im_Y}=-15.3852$, $\frac{\partial r}{\partial c_{ei}}=-0.0005420$,

$\frac{\partial r}{\partial IM_0}=-0.0001186$

(98) $\frac{\partial r}{\partial ex_Y}=15.3852$, $\frac{\partial r}{\partial c_{ee}}=0.0005420$, $\frac{\partial r}{\partial EX_0}=0.0001186$, $\frac{\partial r}{\partial r_{CH}}=5088.0463$, $\frac{\partial r}{\partial CH_0}=2.5263$

(99) $\frac{\partial r}{\partial c_{TF}}=16.3443$, $\frac{\partial r}{\partial TF_0}=0.0001260$, $\frac{\partial r}{\partial t_Y}=-12.0093$, $\frac{\partial r}{\partial TR_0}=-0.00009261$, $\frac{\partial r}{\partial md_Y}=38.3384$

(100) $\frac{\partial r}{\partial md_r}=-0.0004542$, $\frac{\partial r}{\partial m_S}=-0.5954$, $\frac{\partial r}{\partial M_0}=-0.0002956$

From these values we can obtain the following conclusions:

- An increase of the marginal propensity to consume with 0.01 will give a GDP growth of 1023 mil. lei 2000, but also an increase of the real interest rate with an absolute value 0.15%;
- An increase of the marginal index of final consumption of the government with 0.01 will give a GDP growth of 362 mil. lei 2000 and also a non significant increase of the real interest rate with an absolute value 0.05%;
- An increase of the marginal index of other revenues with 0.01 will give a GDP growth of 298 mil. lei 2000 and also a non significant increase of the real interest rate with an absolute value 0.04%;
- An increase of the rate of investments with 0.01 will give a GDP growth of 1056 mil. lei 2000, but also an increase of the real interest rate with an absolute value 0.15%;
- An increase of the factor of influence on the investment rate with 100 will give a GDP decrease of 125 mil. lei 2000 and also a non significant decrease of the real interest rate with an absolute value 0.01%;
- An increase of the rate of imports with 0.01 will give a GDP decrease of 1056 mil. lei 2000 and also a decrease of the real interest rate with an absolute value 0.15%;
- An increase of the factor of imports influence on the exchange rate with 1000 will give a GDP decrease of 3721 mil. lei 2000 and also a significant decrease of the real interest rate with an absolute value 0.5%;
- An increase of the rate of exports with 0.01 will give a GDP increase of 1056 mil. lei 2000 and also an increase of the real interest rate with an absolute value 0.15%;
- An increase of the factor of exports influence on the exchange rate with 1000 will give a GDP increase of 3721 mil. lei 2000 and also a significant increase of the real interest rate with an absolute value 0.5%;
- An increase of the marginal index of the exchange rate according to time with 0.0001 will give a GDP increase of 3494 mil. lei 2000 and also an increase of the real interest rate with an absolute value 0.5%;
- An increase of the marginal index of government transfers according to the output with 0.01 will give a GDP increase of 1122 mil. lei 2000 and also an increase of the real interest rate with an absolute value 0.16%;
- An increase of the marginal index of tax rate according to the output with 0.01 will give a GDP decrease of 825 mil. lei 2000 and also a decrease of the real interest rate with an absolute value 0.12 %;
- An increase of the rate of money demand in the economy with 0.01 will give a GDP decrease of 481 mil. lei 2000 and also an increase of the real interest rate with an absolute value 0.38 %;
- An increase of the factor of influencing the demand for currency from the interest rate with 1000 will give a GDP increase of 569 mil. lei 2000 and also a decrease of the real interest rate with an absolute value 0.4 %;
- An increase of the marginal index of the money supply according to time with 10 will give a GDP increase of 7464 mil. lei 2000 and also a very high decrease of the real interest rate with an absolute value 5%;

Now, for the dinamics model with A= -0.14265, B= -0.00031 computed at averages of ratios $\frac{\frac{dY}{dt}}{D(t)-Y(t)}$, $\frac{\frac{dr}{dt}}{MD(t)-MS(t)}$ in the given period we have:

$$(101)\quad \begin{cases} Y(t)=1062.64-0.5274t+(1.0507C_1-0.0001926C_2)e^{0.852t}+ \\ \quad (-0.0507C_1+0.0001926C_2)e^{0.182t} \\ r(t)=2970.64t-5835602.49+(276.804C_1-0.0507C_2)e^{0.852t}+ \\ \quad (-276.804C_1+1.0507C_2)e^{0.182t} \end{cases}$$

$C_1, C_2 \in \mathbf{R}$

7 Conclusions

The model presented above shows a more flexibility in macroeconomic modeling, because it removes the common assumptions of constancy of variables. Thus, imports, exports, government consumption, transfers etc. are approached by their econometric dependence of GDP and other variables.

Romania's situation, presented in the case study, reveals a contradictory economic policy, in 2004-2008, the Romanian economy being overheated.

Recent years (2013-2014) approached the interest rate and GDP from equilibrium, which was reflected in an dynamic increased of investments.

For Romania, the analysis of marginal indicators proposes as directions for growth: the increase of investments, net exports, government consumption marginal, but also a diminishing of the marginal propensity to consume.

We can estimate a prognosis for 2015 in order to verify the validity of the model.

The value for equilibrium for GDP in 2015 is: 132647.1 lei 2000. Because in the last period (2008-2014) the ratio between the real GDP and that of the equilibrium was approximately constant – 104.55% we obtain an estimated value: 138677 lei 2000. Because the cumulative deflator between 2000 and 2015 is 0.1940 we obtain a prognosis: Y(2015)=714829.7 lei. The real value (estimated at the beginning of 2016) is 712932 lei therefore an error: 0.27%.

We can conclude after this that the model verify well the real data.

4 References

Beaud Michel, Dostaler Gilles, La pensée Économique depuis Keynes, Édition du Seuil, 1996

Bergin Paul R., How well can the new open economy macroeconomics explain hte exchange rate and current account?, NBER Working Paper No.10356, 2004

Gali Jordi, The return of the Phillips curve and other recent developments in business cycle theory, Spanish Economic Review, Springer-Verlag 2000

Hahn, Frank Horace, Keynesian Economics and General Equilibrium Theory: Reflections on Some Current Debates, in Microeconomic Foundations of Macroeconomics edited by Harcourt, London, 1977, pp. 25-40

Hansen A.H., A Guide to Keynes, 1959, Mc Graw-Hill

Hicks J.R., Mr. Keynes and the “Classics”; A Suggested Interpretation, Econometrica, Vol. 5, No. 2, 1937, pp. 147-159

Justiniano Alejandro, Preston Bruce, Can Structural Small Open Economy Models Account for the Influence of Foreign Disturbances?, NBER, Working Paper No.14547, December, 2008

Justiniano Alejandro, Preston Bruce, Monetary policy and uncertainty in an empirical small open-economy model, Journal of Applied Econometrics, Volume 25, Issue 1, January, 2010, pp. 93-128

King Robert G., Will the New Keynesian Macroeconomics Resurrect the IS-LM Model?, Journal of Economic Perspectives, Volume 7, Number 1 (Winter 1993), pp. 67-82

Lawn Philip A., Environmental Macroeconomics: Extending the IS-LM Model to Include an 'Environmental Equilibrium' Curve, Australian Economic Papers, 2003, vol. 42, issue 1, pp.118-134

Lawn Philip A., On Heyes’ IS–LM–EE proposal to establish an environmental macroeconomics, Environment and Development Economics 8: 31–56, 2003 Cambridge University Press

Martínez-García Enrique, Vilán Diego, Bayesian Estimation of NOEM Models: Identification and Inference in Small Samples, Federal Reserve Bank of Dallas, Globalization and Monetary Policy Institute, Working Paper No. 105, March 2012

Romer David., Advanced Macroeconomics, 1996, McGraw-Hill

Romer David, Keynesian Macroeconomics without the LM Curve, Journal of Economic Perspectives, Volume 14, Number 2 (Spring 2000) Pages 149–169

Schmitt-Grohe Stephanie, Uribe Martin, Closing small open economy models, Journal of International Economics 61, March, 2002, pp. 163–185

Smith, R.P.; Zoega, G., Keynes, investment, unemployment and expectations, International Review of Applied Economics, 2009, 23(4), pp. 427-444

Weerapana, Akila, Intermediate macroeconomics without the IS-LM model, Journal of Economic Education 34. 3 (Summer 2003), pp.241-262