Luo, Jiawen; Çepni, Oğuzhan; Demirer, Rıza et al.

Book

Forecasting multivariate volatilities with exogenous predictors : an application to industry diversification strategies

University of Pretoria
Department of Economics Working Paper Series

Forecasting Multivariate Volatilities with Exogenous Predictors: An Application to Industry Diversification Strategies

Jiawen Luo
South China University of Technology
Oğuzhan Çepni
Copenhagen Business School
Riza Demirer
Southern Illinois University Edwardsville
Rangan Gupta
University of Pretoria
Working Paper: 2022-58
December 2022

Department of Economics
University of Pretoria
0002, Pretoria
South Africa
Tel: +27 12 420 2413

Forecasting Multivariate Volatilities with Exogenous Predictors: An Application to Industry Diversification Strategies

Jiawen Luo [*], Oguzhan Cepni [**], Riza Demirer [***] and Rangan Gupta [****]

Abstract: We propose a procedure to forecast the realized covariance matrix for a given set of assets via spectral decomposition within a multivariate heterogeneous autoregressive (MHAR) framework. Utilizing high-frequency data for the U.S. aggregate and industry indexes and a large set of exogenous predictors that include financial, macroeconomic, sentiment, and climate-based factors, we evaluate the out-of-sample performance of industry portfolios constructed from forecasted realized covariance matrices across various univariate and multivariate forecasting models. While the climate and sentiment-based forecasting models generally yield more accurate forecasts of realized covariance compared to the macroeconomic and financial based models, particularly at the short forecast horizon, we find that the models that include industry-level information, generally yield better economic outcomes, in line with the established evidence of the predictive information captured at the industry level. Our results suggest that the MHAR framework coupled with DRD decomposition that splits the covariance matrix into a diagonal matrix of realized variances and realized correlations, can be utilized in a high-frequency setting to implement diversification and smart beta strategies for various investment horizons; however, the choice of the predictors should be aligned with the target investment horizon.

JEL Codes: C32, C53, G10, G11
Keywords: Volatility forecasting, Multivariate HAR model, Forecast evaluation, Beta forecasting, Economic analysis.

[*] Corresponding author. School of Business Administration, South China University of Technology, Guangzhou, China. Email: 2jialuo@163.com.
[**] Copenhagen Business School, Department of Economics, Porcelænshaven 16A, Frederiksberg DK-2000, Denmark. Email: oce.eco@cbs.dk
[***] Department of Economics and Finance, Southern Illinois University Edwardsville, Edwardsville, IL 62026-1102, USA. Email: rdemire@siue.edu.
[****] Department of Economics, University of Pretoria, Private Bag X20, Hatfield 0028, South Africa. Email: rangan.gupta@up.ac.za.

1. Introduction

Accurate forecasts of asset volatility and covariances are critical to maximize diversification benefits, as unstable risk parameters might lead portfolio managers to overstate the benefits of diversification during periods when diversification is needed most (e.g., Ang & Chen, 2002). Accordingly, a large strand of the literature in empirical finance and econometrics has examined a wide array of univariate and multivariate models to forecast return volatility in financial markets.[1] A major challenge in the implementation of these predictive models, however, is the wide diversity of exogenous variables that can be employed as volatility predictors, including macroeconomic and financial (Nonejad, 2017; Paye, 2012) and sentiment-based variables (e.g., Jin et al., 2020; Liang et al., 2020; Wang et al., 2020) in addition to various uncertainty and/or stress indices (Gupta et al., 2014; Singh, 2016; Sum, 2014). In practical applications, however, predictive models that incorporate multiple predictors are often plagued by identification and/or convergence issues (Asgharian et al., 2013; Li et al., 2022). Furthermore, the large majority of these forecasting applications focus on volatility models without extending the analysis to forecasting covariances that are critical in asset allocation strategies.

In this paper, we propose a procedure to forecast the realized covariance matrix for a given set of assets via spectral decomposition within a multivariate heterogeneous autoregressive (MHAR) framework. The procedure adopts a DRD decomposition approach whereby we split the covariance matrix into a diagonal matrix of realized variances D and realized correlations R that are forecasted separately. Within this framework, we construct a set of multivariate volatility models by introducing various groups of exogenous predictors associated with financial, macroeconomic, investor sentiment and climate related factors as well as cross-industry information emanating from industry-level returns. To select the important predictors for the optimal specification, we employ the least absolute shrinkage and selection operator (LASSO), which allows us to reduce the dimension of predictors and eliminate the multicollinearity problem. The forecasted realized volatilities and the correlation matrices are

[1] See Rapach et al., 2008; Rangel and Engle, 2011; Engle et al., 2013 for a detailed discussion of research in this area.

then used to compute the forecasted realized covariance matrices for a given set of assets included in the multivariate model. Finally, the economic efficacy of this procedure is tested within an industry diversification framework by examining the economic outcomes from minimum-variance and beta-neutral investment strategies. Undoubtedly, this is of high interest to investors in forward-looking strategies that aim to achieve maximum diversification in investment portfolios.

Given the importance of volatility as a key risk parameter in financial applications, a large number of studies in the literature has deployed a wide range of volatility forecasting models to assess the risk of financial assets (see, e.g., Wang et al., 2015; Wang et al., 2016; Bee et al., 2016; Clements & Liao, 2017; Li & Wei, 2018; Clements & Preve, 2021; Ji et al., 2022; Golosnoy & Gribisch, 2022), price financial derivatives (Graham & Harvey, 1996; Zhu & Ling, 2015; Symitsi et al., 2018), and compute asset allocations in portfolio optimization models (Ayub et al., 2015; Cederburg et al., 2020). In these applications, a wide range of predictors has been employed, without yielding a consensus on the performance of these predictors in terms of their ability to produce accurate forecasts. A common shortcoming of these applications, however, is that the economic implications of the forecasts obtained from these models have generally been understudied, while extending the forecasting models to asset covariances has largely been ignored despite the fact that accurate covariance forecasts play a critical role in portfolio allocation strategies. To this end, the proposed multivariate HAR framework that adopts the so-called DRD decomposition approach presents a valuable opening to extend these works to forecasting co-movement dynamics as it allows us to model volatility in a multivariate setting. In this paper, building on the extensive literature on forecasting volatility, we test the forecasting efficacy of a large set of exogenous predictors, including financial, macroeconomic, sentiment, and climate-based factors. This allows us to compare the forecasting performance of various multivariate forecasting models that employ different types of predictors and thus offer a novel perspective on the predictive information captured by these predictors at the short and long forecast horizons along with its economic implications.

Indeed, our analysis shows that the LASSO-based multivariate HAR models that employ climate and sentiment-based predictors generally yield more accurate forecasts of realized

covariances compared to forecasting models that employ macroeconomic and financial predictors, in line with the established evidence that sentiment (e.g. Baker and Wurgler, 2016) and climate (Faccini et al., 2021) related factors significantly drive return and volatility dynamics in financial markets. At the same time, forecasting models that employ industry-level information are also found to yield comparable forecasting gains, particular at longer forecat horizons, supporting the gradual diffusion of information hypothesis of Hong et al. (2007, 2014) that the information contained in industry returns diffuses gradually across markets, thus leading to a lead-lag relationship between industry and aggregate market volatility patterns.

As another novelty of our study, we extend our forecasting analysis to an economic context by adopting an asset allocation perspective and examining the economic significance in two different contexts. The first adopts an industry diversification perspective wherein we construct minimum variance industry portfolios using the industry allocations obtained from the realized covariance forecasts generated by each MHAR model. Specifically, the time series of the realized covariance matrix forecasts obtained from each competing forecasting model are extracted, and industry portfolio weights are computed within a minimum variance portfolio setting. Interestingly, our findings show that while the climate-based forecasting models generally yield the lowest volatility in industry diversified portfolios, particularly at the short forecast horizon, the most effective industry diversification outcomes are obtained through models that employ cross-industry information as predictors, once again, in line with the established evidence of the predictive information captured at the industry level. These results suggest that the choice of predictors employed in forecasting models should align with the diversification strategy adopted by investors.

In the second economic approach, we focus on the realized industry betas based on the forecasted variance-covariance elements of the S&P 500 index and industry indices obtained from competing MHAR models. Using these forecasted industry betas, we construct beta-neutral industry portfolios and compare the actual betas of these portfolios to evaluate the efficacy of the forecasting models to attain beta neutrality at various forecast horizons. Once again, our results show that multi-variance models incorporating industry level information as predictors generally yield the lowest portfolio betas, particularly at the short and intermediate

forecast horizons. This suggests that beta forecasts obtained via forecasting models that employ past industry information can help achieve beta neutral portfolios for diversified investors more effectively than the other forecasting models that incorporate financial, macroeconomic, sentiment and climate-based factors. Overall, our results show that the MHAR framework coupled with DRD decomposition that splits the covariance matrix into a diagonal matrix of realized variances and correlations can be successfully utilized in a high frequency setting in order to implement diversification and smart beta strategies for various investment horizons. Our findings have significant implications for portfolio optimization and risk forecasting.

The remainder of the paper is organized as follows. Section 2 describes the data, exogenous predictors used in the forecasting models, and the various multivariate MHAR models employed in our empirical application. Section 3 presents empirical findings from the evaluation of the forecasting performance of the proposed MHAR models and the economic analysis based on various industry diversification strategies. Finally, Section 4 concludes the paper with the investment implications of the findings and directions for future research.

2. Data and Methodology

2.1 Data

We collect 5-minute interval intraday data for the S&P 500 index and its ten industry sub-indices, namely: Information technology (INFT), Health care (HLTH), Consumer discretionary (COND), Communication services (TELS), Financials (FINL), Industrials (INDU), Consumer staples (CONS), Materials (MATR), Utilities (UTIL), Energy (ENRS). The choice of 5-minute frequency to compute the realized measures used in our subsequent tests follows the argument by Ait-Sahalia et al. (2005) and Andersen et al. (2011) that 5-minute frequency can achieve a balance between measurement accuracy and microstructure noise. The intraday price data, obtained from Bloomberg, is available in a continuous format and covers the period of January 3, 2011 and November 29, 2019, where the sample starting date is driven by data availability. As explained later, the intraday data is used to compute the realized volatility estimates at the daily frequency.

In addition to the intraday data, we collect data on four groups of exogenous predictors that

cover broad categories of financial, macroeconomic and sentiment-based indicators in addition to proxies of climate risk that are shown to serve as a systematic driver of stock market returns in recent studies (e.g. Bolton and Kacperczyk, 2021; Cepni et al., 2022).[2] Categorizing the exogenous predictors into separate groups allows us to examine the predictive performance of each group of predictors against others in our subsequent tests. In the first group, we consider various financial market related variables as predictors of realized volatility and covariances. These variables include various volatility indexes in addition to the Goldman Sachs financial conditions index and the risk aversion index of Bekaert et al. (2022), which is shown to be associated with equity variance risk premiums. The second group includes macroeconomic variables, including the Morgan Stanley shadow short rate, which is a measure that captures monetary policy actions, the US economic policy uncertainty index of Baker et al. (2016), along with various measures of economic activity, including the BDI Baltic Exchange Dry Index, Aruoba Diebold Scotti Business Conditions Index and the Goldman Sachs Economic Surprise Index.

In the third group, considering the established evidence in the behavioral finance literature that suggests investors' over or underreaction to news regarding fundamentals drive stock market returns and volatility (Baker and Wurgler, 2006; Tetlock, 2007), we include various proxies of investor sentiment obtained either by textual analysis of news articles or computed from historical price and volume patterns. Finally, in the fourth group of predictors, building on the evidence from Choi et al. (2020) that investors are likely to focus on climate risk-related issues and react to these events rather than fundamental information, we consider several climate-based indicators compiled by Faccini et al. (2021) via textual and narrative analysis of Reuters climate change news. Specifically, this group includes daily measures of climate uncertainty associated with the occurrence of natural disasters, global warming, U.S. climate policy (actions and debate), and international summits on climate-change. In addition to these four climate risk proxies, we also include a narrative U.S. climate policy factor obtained by performing a narrative analysis on news content associated with the climate policy. Table A2 in the Appendix presents the descriptive statistics of the exogenous predictors used in the

[2] The complete list of the variables for the four groups of exogenous predictors is presented in Table A1.

forecasting models discussed next.

2.2 Volatility estimators

Given the multivariate nature of the forecasting models, we forecast the realized volatilities and realized correlations for the industry sub-indices as well as the aggregate market index separately. For each index *i*, the realized volatility (*RV*) is computed as

$$RV_{i,t+1}(\delta) = \sum_{j=1}^{1/\delta} r_{i,t+j\delta,\delta}^2 \tag{1}$$

where δ is the sample frequency of the realized volatility and $r_{i,t+j\delta,\delta}$ is the 5-minute frequency returns calculated by $r_{i,t+j\delta,\delta} = 100 \times (log\, P_{i,t+j\delta} - log\, P_{i,t+(j-1)\delta})$. Likewise, the realized covariance is computed as:

$$RV_{t+1}(\delta) = \sum_{j=1}^{1/\delta} \boldsymbol{r}_{t+j\delta,\delta} \boldsymbol{r}'_{t+j\delta,\delta} \tag{2}$$

where $\boldsymbol{r}_{t+j\delta,\delta}$ is a vector of returns of the S&P 500 and 10 industry indices. The realized correlation is computed by the spectral decomposition method as

$$RC_t = SD_t^{-1} * RCOV_t * SD_t^{-1\prime} \tag{3}$$

where $SD_t = diag\{RV_t, \ldots RV_t\}$ is the matrix of standard deviations.

The descriptive statistics for daily-realized volatilities of the S&P 500 index and 10 industry indices are shown in Table 1. The mean realized volatility estimates range between a high of 1.07% for the energy sector (ENRS) and a low of 0.34% for consumer staples (CONS). Interestingly, consumer staples sector experiences even lower realized volatility than the aggregate market index (SPX) with a mean RV value of 0.44%. The high kurtosis values observed for all RV series indicate the presence of extreme fluctuations in intraday return patterns.

[Insert Table 1 here]

The time series plots of estimated realized volatility series presented in Figure 1 display notable spikes in volatility in mid to late 2011 when global stock markets suffered heavy losses due to contagion fears emanating from the European sovereign debt crisis and the credit rating downgrade because of the debt-ceiling crisis in the United States. Likewise, a similar uptick in realized volatility is observed in late 2014 when the U.S. military began its airstrike campaign against ISIL in Syria and northern Iraq. While the realized correlations presented in Figure 2 do

not exhibit such notable spikes as observed for realized volatilities, we observe notable drops in realized correlation estimates in mid-2014, most notably in the case of the correlations of information technology, consumer discretionary, financials and industrials with the aggregate market index. Again, this period coincides with the U.S. involvement in the campaign against ISIL, suggesting that geopolitical uncertainty has had a significant impact on the realized correlations of certain industries with the aggregate market.

[Insert Figure 1 here]

[Insert Figure 2 here]

2.3 Multivariate forecast models within the MHAR framework

As noted earlier, the novelty of our analysis is a framework wherein we forecast the realized covariance matrix of intraday industry returns via spectral decomposition within a multivariate heterogeneous autoregressive (MHAR) framework. In this regard, we construct a set of multivariate volatility models by introducing various groups of exogenous predictors associated with financial, macroeconomic, investor sentiment, and climate related factors as well as cross-industry information emanating from the other industries. To select the important predictors to identify the optimal specification, we also introduce a LASSO approach, which allows us to reduce the dimension of predictors and eliminate the multicollinearity problem. The procedure adopts a DRD decomposition whereby we split the covariance matrix into a diagonal matrix of realized variances D and realized correlations R that are forecasted separately. The forecasted realized volatilities and the correlation matrix are then used to compute the forecasted realized covariance matrices for a given set of assets included in the multivariate model.

Specifically, assuming that the correlations across the industries are relative stable across time, we approximate the forecasting correlation matrix $\widehat{RC}_{t+1|t}$ with the moving average of the realized correlation matrices over the past *N* trading days.

$$\widehat{RC}_{t+1|t} = \frac{1}{N}\sum_{n=1}^{N} RC_{t+1-n} \tag{4}$$

where we set *N=T1* so that the forecasted correlation matrix $\widehat{RC}_{t+1|t}$ is the moving average of the realized correlation matrices over a rolling window of in-sample period *T1*

days.[3]

Given the forecasted realized volatilities, we then obtain the standard deviation matrix such that $\hat{S}D_{t+1:t+h|t} = diag\{\sqrt{\widehat{RV}_{1,t+1:t+h|t}}, \ldots \sqrt{\widehat{RV}_{N,t+1:t+h|t}}\}$ and compute the covariance matrix for h-step forecast horizon according to the spectral decomposition method as

$$\hat{\Sigma}_{t+1:t+h|t} = \hat{S}D_{t+1:t+h|t} * \widehat{RC}_{t+1|t} * \hat{S}D_{t+1:t+h|t}{}' \tag{5}$$

where $\hat{\Sigma}_{t+1:t+h|t}$ is the h-step forecast realized covariance matrix.

In our empirical analysis, we consider eight different heterogeneous autoregressive (HAR) model specifications with various predictor sets to forecast the realized volatilities. The benchmark model is set as the traditional HAR model proposed by Corsi (2009) that includes the daily, weekly and monthly lagged volatilities to generate forecasts of realized volatility. Next, using the forecasted realized correlations volatilities, we construct eight types of multivariate models to forecast the realized covariance of the aggregate stock market and the industry indices, which we term as the HAR-DRD models. The HAR-DRD specifications differ from the HAR type models used to generate forecasts of realized volatilities. The first model, termed the HAR model, is the benchmark HAR model that includes only the lagged daily, weekly, and monthly lagged realized volatilities and is formulated as

$$\log(\mathrm{RV}_t) = \beta_0 + \beta_1 \log(\mathrm{RV}_{t-1}) + \beta_2 log(\mathrm{RV}_{t-1:t-5}) + \beta_3 \log(\mathrm{RV}_{t-1:t-22}) + u_t \tag{6}$$

The HAR-Macro, HAR-Finance, HAR-Sentiment and HAR-Climate models incorporate the macroeconomic, financial, sentiment and climate-based predictors, respectively and are formulated respectively in Equations 7-10 as

$$\mathbf{HAR-Macro}: \log(\mathrm{RV}_t) = \beta_0 + \beta_1 \log(\mathrm{RV}_{t-1}) + \beta_2 log(\mathrm{RV}_{t-1:t-5}) + \beta_3 \log(\mathrm{RV}_{t-1:t-22}) + \sum_{i=1}^{N_1} \alpha_i^d X_{i,t-1}^{macro} + \sum_{i=1}^{N_1} \alpha_i^w X_{i,t-1:t-5}^{macro} + \sum_{i=1}^{N_1} \alpha_i^m X_{i,t-1:t-22}^{macro} + u_t \tag{7}$$

$$\mathbf{HAR-Finance}: \log(\mathrm{RV}_t) = \beta_0 + \beta_1 \log(\mathrm{RV}_{t-1}) + \beta_2 log(\mathrm{RV}_{t-1:t-5}) + \beta_3 \log(\mathrm{RV}_{t-1:t-22}) + \sum_{i=1}^{N_2} \alpha_i^d X_{i,t-1}^{\mathrm{finance}} + \sum_{i=1}^{N_2} \alpha_i^w X_{i,t-1:t-5}^{\mathrm{finance}} + \sum_{i=1}^{N_2} \alpha_i^m X_{i,t-1:t-22}^{finance} + u_t \tag{8}$$

$$\mathbf{HAR-Sentiment}: \log(\mathrm{RV}_t) = \beta_0 + \beta_1 \log(\mathrm{RV}_{t-1}) + \beta_2 log(\mathrm{RV}_{t-1:t-5}) +$$

[3] For the robustness checks, we also use the moving average of the realized correlation matrix over the past 126 days (half year), 252 days (1 year) and 504 days (two years) as the forecasted realized correlation matrix, which yields similar results. These additional results are available in an online appendix due to space considerations.

$$\beta_3 \log(\mathrm{RV}_{t-1:t-2}) + \sum_{i=1}^{N_3} \alpha_i^d X_{i,t-1}^{\mathrm{sent}} + \sum_{i=1}^{N_3} \alpha_i^w X_{i,t-1:t-5}^{\mathrm{sent}} + \sum_{i=1}^{N_3} \alpha_i^m X_{i,t-1:t-22}^{\mathrm{sent}} + u_t \quad (9)$$

$$\mathbf{HAR-Climate}: \log(\mathrm{RV}_t) = \beta_0 + \beta_1 \log(\mathrm{RV}_{t-1}) + \beta_2 log(\mathrm{RV}_{t-1:t-5}) +$$

$$\beta_3 \log(\mathrm{RV}_{t-1:t-22}) + \sum_{i=1}^{N_4} \alpha_i^d X_{i,t-1}^{\mathrm{climate}} + \sum_{i=1}^{N_4} \alpha_i^w X_{i,t-1:t-5}^{\mathrm{climate}} + \sum_{i=1}^{N_4} \alpha_i^m X_{i,t-1:t-22}^{\mathrm{climate}} + u_t \quad (10)$$

where $X_{i,t-1}^{\mathrm{finance}}$, $X_{i,t-1}^{\mathrm{macro}}$, $X_{i,t-1}^{\mathrm{sent}}$ and $X_{i,t-1}^{\mathrm{climate}}$ are the exogenous predictors, including the financial, macroeconomic, sentiment, and climate-based predictors, respectively.

The sixth model, termed the HAR-Volatility model, includes the cross-market information contained in the volatilities of the industry indices and is formulated as

$$\mathbf{HAR-Volatility}: \log(\mathrm{RV}_t) = \beta_0 + \beta_1 \log(\mathrm{RV}_{t-1}) + \beta_2 log(\mathrm{RV}_{t-1:t-5}) +$$

$$\beta_3 \log(\mathrm{RV}_{t-1:t-22}) + \sum_{i=1}^{N_5} \alpha_i^d X_{i,t-1}^{\mathrm{volatility}} + \sum_{i=1}^{N_5} \alpha_i^w X_{i,t-1:t-5}^{\mathrm{volatility}} + \sum_{i=1}^{N_5} \alpha_i^m X_{i,t-1:t-2}^{\mathrm{volatility}} + u_t \quad (11)$$

where $X_{i,t-1}^{\mathrm{volatility}}$ denote the realized volatilities of industry indices. Finally, the last two models, HAR-ALL and HAR-Factor, include respectively, all exogenous predictors used in the previous model variations and the first principle factor of each exogenous predictor set, formulated as

$$\mathbf{HAR-ALL}: \mathrm{RV}_t = \beta_0 + \beta_1 \mathrm{RV}_{t-1} + \beta_2 \mathrm{RV}_{t-1:t-5} + \beta_3 \mathrm{RV}_{t-1:t-22} + \sum_{i=1}^{N_6} \alpha_i^d X_{i,t-1}^{all} +$$

$$\sum_{i=1}^{N_6} \alpha_i^w X_{i,t-1:t-5}^{all} + \sum_{i=1}^{N_6} \alpha_i^m X_{i,t-1:t-22}^{all} + u_t \quad (12)$$

$$\mathbf{HAR-Factor}: \log(\mathrm{RV}_t) = \beta_0 + \beta_1 \log(\mathrm{RV}_{t-1}) + \beta_2 log(\mathrm{RV}_{t-1:t-5}) + \beta_3 \log(\mathrm{RV}_{t-1:t-22}) +$$

$$\sum_{i=1}^{N_7} \alpha_i^d X_{i,t-1}^{\mathrm{factor}} + \sum_{i=1}^{N_7} \alpha_i^w X_{i,t-1:t-5}^{\mathrm{factor}} + \sum_{i=1}^{N_7} \alpha_i^m X_{i,t-1:t-22}^{\mathrm{factor}} + u_t \quad (13)$$

where $X_{i,t-1}^{\mathrm{all}}$ and $X_{i,t-1}^{\mathrm{factor}}$ correspond to the vector of all predictors and the first principal component factor of each predictor set. In these models, α_i^d, α_i^w *and* α_i^m are the coefficient vectors for the daily, weekly and monthly predictor sets and $(N_1, N_2, \ldots, N_6)$ denote the numbers of exogenous predictor sets respectively. It must be noted that the weekly and the monthly predictors are computed by taking the average of lagged realized volatility estimates over the past 5 days and 22 days, respectively, as

$$\mathrm{X}_{t-1:t-N} = \frac{1}{N} \sum_{i=1}^{N} X_{t-i}. \quad (14)$$

Finally, the HAR models above can be written into a general linear regression model as

$$y_t = Z_t \beta + u_t, u_t \sim N(0, \sigma^2) \quad (15)$$

where N(.) is the Gaussian distribution and $\sigma^2$is the variance of the residuals. As there are many predictors in each forecasting model, in order to reduce the number of predictors for a parsimonious specification in each case, we use the variable selection approaches to identify the important predictors. The variable selection approach is motivated by the consideration that (i) including a large number of predictors in the model leads to the greater costs due to greater need for information processing capacity (Gabaix, 2014; Luo and Young, 2016; Zhang, Ma and Wang, 2019) and (ii) several exogenous predictors, such as macroeconomic and financial market indicators, are highly correlated, which in turn can lead to the over-fitting problem. Thus, employing variable selection methods can help to mitigate the risk of over-fitting and improve forecast accuracy (Campbell and Thompson, 2008; Korobilis, 2017; Koop and Korobilis, 2018). To that end, we combine the least absolute shrinkage and selection operator (Lasso), and elastic net variable selection approaches with the HAR models to construct the volatility forecast models. Particularly, the package 'glmnet' of Friedman et al. (2010) is employed to compute the coefficients for Lasso-HAR models by solving

$$\hat{\gamma}^{Lasso} = argmin_{\gamma_0,\gamma}\{\frac{1}{2T}\sum_{t=1}^{T}(y_{i,t} - \gamma_0 - Z_{t-1}\gamma)^2 + \lambda\sum_{j=1}^{K}|\gamma_j|\} \quad (16)$$

where λ is a nonnegative regularization parameter selected given the largest value of lambda such that error is within one standard error of the minimum. Accordingly, we construct 25 different models by introducing various combinations of exogenous predictor sets and combine the Lasso method for predictor selection. The detailed specifications of the 25 HAR-DRD model variations are shown in Table A3 in the Appendix.

3. Empirical Results

3.1 Performance of out-of-sample forecasts

We begin our analysis by examining the out-of-sample forecasts for the realized covariances of the aggregate market and industry indices derived from the competing models described earlier. To evaluate the out-of-sample forecast performances of competing forecast models, we divide the sample into the in-sample and out-of-sample periods, using 2/3 and 1/3 of the entire sample of observations, respectively. The rolling forecasting method is then used to generate short-term (*h=1*), mid-term (*h=5*) and long-term (*h=22*) out-of-sample forecasts, corresponding to one-day, one-week and one-month ahead forecasts. The rolling window is

specified as 2/3 of the sample observations. The accuracy of the realized covariance forecasts are evaluated based on the statistical loss functions. Following Anatolyev and Kobotaev (2018), we compare the out-of-sample forecast performance of the competing multivariate models using two types of loss functions. The first is the Root Mean Square Error (RMSE) loss, which is computed for $t = T_1 + 1, \cdots, T$ out-of-sample forecast as

$$e_{t,t+h} = \Sigma_{t:t+h} - \hat{\Sigma}_{t:t+h|t} \tag{17}$$

$$L^{RMSE} = \frac{1}{T - T_1} \sum_{t=T_1+1}^{T} \sqrt{\sum_{i,j} | e_{t_{ij}} |^2} \tag{18}$$

where $\Sigma_{t:t+h}$ is the volatility proxy, and $\hat{\Sigma}_{t:t+h|t}$ is the h-step ahead covariance matrix forecast. We use the multivariate realized kernel as a proxy for volatility.

The second statistical criterion used to evaluate the forecast performance is the Stein loss function (James and Stein, 1992) formulated as

$$L^{S}_{t+h} = tr(\hat{\Sigma}^{-1}_{t:t+h|t} \Sigma_{t:t+h}) - \log(\hat{\Sigma}^{-1}_{t:t+h|t} \Sigma_{t:t+h}) - k \tag{19}$$

where k is the dimension of the covariance matrix. Note that the Stein function is a multivariate extension of traditional QLIKE loss function.

Additionally, the Model Confidence Set (MCS) developed by Hansen, Lunde, and Nason (2011) is employed to test the significance of difference in forecast performances among the volatility models. The MCS procedure aims to select a set of models with the best forecast performances from a set of candidate forecast models $\mathcal{M}_0 = \{\mathcal{M}^i, i = 1, \cdots, \mathrm{M}\}$. Given a confidence level α, we identify the MCS $\widehat{\mathcal{M}}^*_{1-\alpha} = \mathcal{M}_0$, which is included in the models with the best forecast performance from a set of candidate models. Starting from a full set of models $\mathcal{M}_0$, if $H_{0,\mathcal{M}}$ is accepted, we set $\widehat{\mathcal{M}}^*_{1-\alpha} = \widehat{\mathcal{M}}_{1-\alpha}$; otherwise, we use the elimination rule to remove a model from $\widehat{\mathcal{M}}_{1-\alpha}$. We repeat the procedure until no model can further be eliminated and then the surviving set of models belongs to the MCS $\widehat{\mathcal{M}}^*_{1-\alpha}$. We implement the MCS using the stationary bootstrap procedure of Hansen et al. (2011).

Table 2 presents the pairwise comparisons of the out-of-sample forecasts from the

benchmark HARDRD model against the extended model variations based on RMSE and Stein loss functions. The findings for one-step, five-step and 22-step forecasts of realized covariance are presented in separate panels. Interestingly, climate (LASSO-HARDRD-Climate) and sentiment (LASSO-HARDRD-Sentiment) based forecasting models are found to yield higher forecast accuracy based on the RMSE values. This is further supported by the statistically significant MCS_{TR} and MCS_{TSQ} values, indicating a statistically significant differences in the forecast precision of these models against the benchmark HARDRD model based on the range T_R and the semi-quadratic T_{SQ} statistics, respectively.

In the case of the Stein loss function, however, we find that the HAR-Volatility model (LASSO-HARDRD-volatility) that includes the realized volatilities of industry indices provides the most accurate realized covariance forecasts, both at the short and long forecast horizons, further supported by the statistically significant MCS_{TR} and MCS_{TSQ} values. Relating this finding to the evidence in the literature, the predictive content captured by industry level information is indeed consistent with the well-established evidence that suggests a lead-lag relationship between industry and aggregate market volatility patterns (e.g. Demirer et al., 2020). In addition, employing the Lasso method for predictor selection improves the forecast performance of the corresponding HARDRD models. The improvement in the forecasting performance achieved by combining the Lasso methods has been widely addressed in the literature, mainly due to the identifying the important predictors and eliminating the over-fitting problem (Zhang et al., 2020; Christensen et al., 2021).

[Insert Table 2 here]

3.2 Economic Evaluation

Clearly, the performance of a forecasting model is meaningful if the improved forecast accuracy resulting from the model can be exploited to realize economic gains. Therefore, in order to evaluate the economic significance of the realized covariance forecasts obtained from competing multivariate volatility models, we adopt an asset allocation perspective and examine the economic significance in two different contexts. The first adopts an industry diversification perspective wherein we construct an industry portfolio of the ten industry indices using the industry allocations obtained from the realized covariance forecasts. To that end, the time series of the (10x10) realized covariance matrix forecasts obtained from each competing model are

extracted and industry portfolio weights are computed within a minimum variance portfolio setting. In the second approach, we examine the forecasting performance of competing models by constructing a market-neutral portfolio wherein the asset allocations are computed using the forecasted variance-covariance elements of the S&P 500 index and ten industry indices. We present the details of each evaluation approach next.

3.2.1 Industry diversification using realized covariance forecasts

The implementation of the industry diversification approach follows the Global Minimum Variance Portfolio (GMVP) of Kempf and Memmel (2006). The GMVP of the ten industries is obtained based on the optimization problem similar to the Markowitz setting formulated as

$$min_{w_{t+s}}\, \boldsymbol{w}'_{\boldsymbol{t+h}}\, \hat{\Sigma}_{t+1:t+\square|t} \boldsymbol{w}_{\boldsymbol{t+h}} \tag{20}$$

where $\boldsymbol{w}_{\boldsymbol{t+h}}$is a vector with elements $w_{t+1}^{(n)}$ and $\sum_{n=1}^{N} w_{t+1}^{(n)} = 1$ and the weight of GMVP which is denoted as $\boldsymbol{w}_{\boldsymbol{t+h}}^{\boldsymbol{GMV}}$is computed by

$$w_{t+\square}^{GMV} = \frac{\hat{\Sigma}_{t+1:t+\square|t}^{-1} l}{l' \hat{\Sigma}_{t+1:t+\square|t}^{-1} l} \tag{21}$$

where l is a vector with ones.

Given the weight of GMVP, we obtain the expected volatility and return of minimum-variance portfolio based on the GMVP weight and the realized covariance as

$$\sigma^{2}{}_{t+\square}^{GMV} = \boldsymbol{w}_{\boldsymbol{t+h}}^{\boldsymbol{GMV}\prime} \Sigma_{t+1:t+\square} \boldsymbol{w}_{\boldsymbol{t+h}}^{\boldsymbol{GMV}}, \boldsymbol{r}_{\boldsymbol{t+h}}^{\boldsymbol{GMV}} = \boldsymbol{w}_{\boldsymbol{t+h}}^{\boldsymbol{GMV}\prime} r_{t+1:t+\square} \tag{22}$$

Having constructed the industry portfolio, following Callot et al. (2017) and Bollerslev et al. (2018), we then evaluate the economic performance of various forecasting models via various criteria, including the average turnover, portfolio concentration, the average return, the cumulative return, Sharpe ratio and the economic value based on a quadratic utility function with alternative risk aversion rates. The first evaluation criterion is the portfolio turnover ratio (TO). Following DeMiguel et al. (2014), Callot et al. (2017) Bollerslev et al. (2018), the average portfolio turnover over the out-of-sample period is measured by

$$TO = \frac{1}{T_2} \sum_{t=T_1+1}^{T} \sum_{n=1}^{N} \left| w_{t+1}^{(n)} - w_{t+1}^{(n)} \frac{1+r_t^{(n)}}{1+\boldsymbol{w}_t' \boldsymbol{r}_t} \right| \tag{23}$$

where T_1 and T_2 denote the in- and out-of-sample periods, respectively, w_t is a vector with

elements $w_t^{(n)}$ and r_t is a vector with elements $r_t^{(n)}$. The second criterion follows the argument that investors favor more diversified portfolios that involve less extreme portfolio positions and so we examine the average portfolio concentration ratio (CO) formulated according to Bollerslev et al.(2018) as

$$CO = \frac{1}{T_2}\sum_{t=T_1+1}^{T}\sum_{n=1}^{n}(w_t^{(n)})^2 \tag{24}$$

Next, we investigate the average out-of-sample return (net of transaction costs) for the industry portfolios obtained from each forecasting model formulated as

$$r_{pt} = \boldsymbol{w_t'r_t} - cTO_t \tag{26}$$

where the proportional transaction cost is assumed to cTO_t. Following Fleming et al. (2003), Brown and Smith (2011) and Bollerslev et al. (2018), we set three values for c at 0%, 1% and 2% respectively. The average return is then computed by taking the average of the out-of-sample portfolio returns such that $r_p = \frac{1}{T_2}\sum_{t=1}^{T_2} r_{pt}$. For easier comparison, we report the approximately annualized average out-of-sample return compute as $r_p \times 252$.

The next criterion to evaluate the portfolios is the cumulative return. Following Callot et al. (2017), the cumulative return over the out-of-sample is computed as

$$r_p^{Accum} = \prod_{t=1}^{T_2}(1 + r_{pt}) \tag{27}$$

where T_2 denotes the out of sample period. Next, we examine two risk-adjusted performance measures. The first is the out-of-sample Sharpe ratio computed as

$$S\square arpe\ \ ratio = \frac{r_p}{\sigma_p} \tag{28}$$

where the standard deviation of the portfolio σ_p is computed by

$$\sigma_p = \sqrt{\frac{1}{T_2}\sum_{t=T_1+1}^{T}(r_{pt} - \frac{1}{T_2}\sum_{t=T_1+1}^{T} r_{pt})^2} \tag{29}$$

The second risk-adjusted performance measure is the economic value, defined as the value of Δ such that $\sum_{t=T_1+1}^{T} U(r_{pt}^k) = \sum_{t=T_1+1}^{T} U(r_{pt}^l - \Delta)$ for two different portfolios p_1 and p_2 such that the greater the Δ value, the higher the returns that a risk-averse investor would be willing to sacrifice to switch from model l to model k. To this end, following Fleming et al. (2003), Bollerslev et al. (2016), and Callot et al. (2017), we assume a quadratic utility function with risk aversion γ to determine the economic values of various forecast models, formulated as

$$U(r_{pt}, \gamma) = (1 + r_{pt}) - \frac{\gamma}{2(1+\gamma)}(1 + r_{pt})^2 \quad (30)$$

For robustness, we consider two levels of risk aversion to capture various levels of risk preferences for an investor, i.e. the mild (γ=1) and high-risk aversion rate (γ=10).

Table 3 reports the economic evaluation results for the industry portfolios obtained from competing forecasting models for the short (h=1), mid (h=5), and long (h=22) forecast horizons. Not surprisingly, the expected volatility for the GMVP increases with the forecast horizon, although the LASSO-based forecast models generally yield lower volatility consistently across the different model specifications and forecast horizons. Interestingly, once again, the climate-based forecasting model (LASSO-HARDRD-climate) yields the lowest portfolio volatility at the short horizon, highlighting the increasing role of climate uncertainty over market fluctuations. This is consistent with the growing evidence that investors are increasingly aware of climate concerns in investment decisions and price financial assets accordingly (e.g. Bolton and Kacperczyk, 2021). However, our evidence shows that this information is helpful in modeling expected risk, particularly at short forecast horizons. The HAR-Finance model (LASSO-HARDRD-Finance), which incorporates financial market-based predictors, is found to yield the lowest intermediate and long run volatility forecasts across the constructed portfolios.

Regarding returns in excess of transaction costs, sentiment-based models generally outperform the others, particularly at the mid to long forecast horizons. However, the industry-based model (HAR-Volatility) experiences a lower turnover ratio. In terms of risk adjusted returns, the models that include industry level information generally yield higher Sharpe ratios, particularly at the long forecast horizon. We also observe that the HAR-DRD models combined with exogenous variables always have lower turnover than the benchmark model, thus yielding improved portfolio returns based on both transaction cost parameters considered. These models are also found to have lower concentrations ratio compared with the benchmark model, indicating greater portfolio diversification at lower transaction costs. Furthermore, we find that the LASSO-HARDRD-Sentiment model achieves a higher Sharpe ratio and positive economic values against the benchmark models consistently for all the forecast horizons, given the two levels of transaction costs.

Finally, examining the cumulative returns from the portfolios derived from various forecasting models, presented in Figure 4, we observe that the benchmark HARDRD model generally outperforms the other model variations highlighting the informational value of the predictors in portfolio optimization models. We find for the short-term forecasts that the Lasso-HAR-all model outperforms all other variations in terms of accumulated returns until mid-2018, while the HAR-DRD-sentiment model takes the top spot after this period. Interestingly, this sub-period corresponds to the new presidential cycle led by the Trump administration that came into power in 2018, suggesting that the sentiment related predictors captured an increasingly important information value during this period, perhaps due to increased political uncertainty. For the mid-term forecasts, the Lasso-HAR-DRD-all model dominates its counterparts consistently during the entire sample period, while the climate-based portfolio outperforms other models at the long forecast horizon (h-22). Overall, these results highlight that the informational value of the predictors depends on the forecast horizon and suggest that the predictor set to be utilized in forecasting models has to be aligned with the target investment horizon.

[Insert Table 3 here]

[Insert Figure 4 here]

3.2.2 Beta-neutral portfolios based on realized covariance forecasts

Having examined the economic outcomes for the minimum variance strategy across the various forecasting models, we next extend our economic analysis in the context of a diversified portfolio and examine the forecasting performance of competing models by constructing market-neutral portfolios wherein the industry allocations are computed based on the forecasted variance-covariance elements of S&P 500 index and ten industry indices. For comparison purposes, the time series plots for the daily realized industry betas obtained from intraday returns are presented in Figure 3. While the realized industry betas generally hover around unity for most industries, not surprisingly, we observe higher realized betas for information technology (INFT) and Energy (ENRS). In comparison, Utilities (UTIL) generally experience lower betas below unity. Interestingly, there seems to be a structural break in betas for Communication services (TELS) in mid-2018 with a jump in betas above unity, which could be in part due to the data protection rules imposed by the European Union in May 2018 as a

result of the General Data Protection Regulation (GDPR), which in turn has led to several complaints filed against US technology giants along with a blockage of digital content by the EU against several major U.S. news outlets engaged in the European market.

In the next step, having generated the variance-covariance matrix forecasts that include the S&P 500 index and the industry indices based on competing MHAR model specifications listed in Table 4, we first compute daily beta forecasts ($\hat{\beta}_j$) for each industry j as

$$\hat{\beta}_{j,t} = \frac{\widehat{RCOV}_{j,S\&P500}}{\widehat{RV}_{S\&P500}} \tag{31}$$

where the $\widehat{RCOV}_{j,S\&P500}$ is the covariance forecast between the S&P 500 index and industry j and $\widehat{RV}_{S\&P500}$ is the variance forecast for the S&P 500 index, which are extracted from the forecast covariance matrix for a given model. Specifically, we use each competing MHAR model to generate realized covariance matrix forecasts and use elements of the forecasted covariance matrices to generate industry beta forecasts.

Having computed the realized beta series for each industry, we next sort the industries into five portfolios based on their respective beta forecasts. Next, we create beta-neutral portfolios, ex ante, by solving $\upsilon_{j,t}\hat{\beta}_{long\ j,t} - \hat{\beta}_{short,j,t} = 0$ and applying the resulting weight $\upsilon_{j,t}$ to the long side of the portfolio, where $\hat{\beta}_{short,j,t}$ is the average beta for industries in the lowest forecast beta group and $\hat{\beta}_{long,j,t}$ is average beta for industries in the largest forecast beta group. Table 4 presents the actual beta values for the constructed beta-neutral portfolios obtained from the beta forecasts for each model variation for the three forecast horizons under consideration. We also compute the t-statistics according to the robust Newey and West (1987) standard errors with six lags to test whether the actual beta of the portfolio is significantly different from zero. We observe that the HAR-Volatility model that incorporates industry level information in the forecasting model generally yields the lowest beta values for the portfolio, particularly at the short and intermediate forecast horizons. Furthermore, the insignificant t-statistics associated with those beta values in the HAR-Volatility model suggest that the beta forecasts obtained through cross-industry information can help achieve beta neutral portfolios for diversified investors more effectively than the other forecasting models that incorporate financial, macroeconomic, sentiment, and climate-based factors. In short, our results show that beta neutral industry diversification can be best achieved via forecasting models that utilize cross-industry information as predictors. Particularly, the LASSO-HARDRD-volatility and LASSO-HARDRD-volatility models achieve insignificant beta-neutral value for the short-term

forecasts, suggesting that these two models outperform the other models in constructing market-neutral portfolios.

[Insert Table 4 here]

3.3 Robustness Checks

In order to check the robustness of our findings, we conduct a number of additional tests by employing alternative volatility estimators and forecast windows used to generate the realized covariance matrix forecasts and to construct the minimum variance and beta neutral portfolios. First, following Fiszeder and Perczak (2016) and Luo et al. (2019), we conduct a robustness check using the realized kernel estimator of Hansen and Lunde (2006), which corrects the effects of market microstructure noise on the estimation of *RV*. The realized kernel estimator (RK) is formulated as

$$RK_{t+1} = \sum_{j=1}^{n} r_{t,j}^2 + 2\sum_{w=1}^{q}\left(1-\frac{w}{q+1}\right)\sum_{i=1}^{n-w} r_{t,i} r_{t,i+w}$$

Following Hansen and Lunde (2006), we set $q = \left\lceil \frac{\omega}{(b-a)/n} \right\rceil$ where $\lceil x \rceil$ is the smallest integer that is greater than or equal to *x*, ω is the desired width of the lag window, and *b*-*a* is the length of the sampling period.

Table A4 in the Appendix presents the pairwise comparisons of the out-of-sample forecasts from the benchmark HARDRD model against the extended model variations based on RMSE and Stein loss functions using the realized volatility forecasts estimated by the realized kernel estimator. Again, we observe that the climate-based LASSO-HARDRD-climate model yields the lowest forecast error at the short and long forecast horizons, consistent with the evidence presented in Table 2. Similarly, the sentiment-based LASSO-HARDRD-sentiment model performs the best at the intermediate forecast horizon, supporting our earlier results. Further examining the economic evaluation results in Tables A5 and A6 in the Appendix, we observe similar results in that the LASSO-based forecast models generally yield lower volatility. While the industry portfolio constructed based on the realized covariance estimates from the climate-based forecasting model (LASSO-HARDRD-climate) yields the lowest portfolio volatility at the short horizon, we find that the HAR-Finance model (LASSO-HARDRD-Finance) that incorporates financial market-based predictors yields the lowest intermediate and long run volatility forecasts across the constructed portfolios. Similarly, the finance-based model generally outperforms the others in terms of portfolio returns, particularly at the short to mid forecast horizons, while the industry-based model (HAR-Volatility) offers

better return performance at the long forecast horizon. Finally, the analysis of the beta-neutral portfolios presented in Table A6 yields similar results as observed earlier: the models that incorporate cross-industry information (LASSO-HARDRD-volatility) yield the most effective portfolios in terms of achieving beta neutrality in industry portfolios.

In addition to using an alternative volatility estimator as a robustness check, we also replicated our analysis using alternative forecast windows that we used to generate the realized covariance estimates. For this purpose, we considered a 1/3 and 2/3 split of the sample period to be used for the in- and out-of-sample forecasts, respectively. Although not reported in the paper due to space considerations, the additional results for the forecast precision evaluation and the subsequent economic evaluations are provided in Tables OA1- OA2 in an online appendix. Additionally, we examined the robustness of our findings to the time frame used to compute the moving averages for the realized correlation matrices. For this purpose, unlike the original analysis where we used the moving average of the realized correlation matrix over the past T1 days as the forecasted realized correlation matrix, we also examined the moving averages over the past 126 days (half year), 252 days (1 year) as well as the past 504 days (two years). These additional results yield similar conclusions and are presented in Table OA3-OA8 in an online Appendix to save space. We also report the accumulated returns of different models for different forecast horizons given the long out-of-sample period in Figure OA1, which draws similar conclusions with our main results discussed above.

4. Conclusion

This paper proposes a forecasting framework that can be used to generate accurate forecasts for the realized covariance matrix of asset returns via spectral decomposition within a multivariate heterogeneous autoregressive (MHAR) framework. Extending the heterogeneous autoregressive (HAR) model of Corsi (2009), which has now become a useful model for describing realized volatility dynamics, particularly in high frequency applications, to a multivariate context, we examine the predictive performance of various multivariate HAR models that incorporate exogenous predictors associated with financial, macroeconomic, investor sentiment and climate related factors. The procedure adopts a DRD decomposition approach whereby we split the covariance matrix into a diagonal matrix of realized variances D and realized correlations R that are forecasted separately. The forecasted realized volatilities and the correlation matrix are then used to compute the forecasted realized covariance matrices

for a given set of assets included in the multivariate model.

In an application of the proposed framework to an industry diversification context, utilizing intraday data for the S&P 500 index and ten industry indices over the period January 3, 2011 and November 29, 2019, we evaluate the forecasting performance of various multivariate HAR-DRD models based on a wide range of evaluation criteria. We find that the climate (LASSO-HARDRD-Climate) and sentiment (LASSO-HARDRD-Sentiment) based forecasting models generally yield more accurate forecasts of realized covariance compared to the macroeconomic and financial predictors, providing support for the growing evidence that sentiment (e.g. Baker and Wurgler, 2016) and climate (Faccini et al., 2021) related factors significantly drive return and volatility dynamics in financial markets. Further extending our analysis to an industry diversification context, we use the realized covariance matrix forecasts obtained from each competing multivariate HAR model to create (i) minimum variance and (ii) beta-neutral industry portfolios. While the climate-based forecasting model is found to yield the lowest portfolio volatility, particularly at the short forecast horizon, we find that the models that include industry level information generally yield higher risk-adjusted returns, in line with the established evidence of the predictive information captured at the industry level. Finally, extending the analysis to beta-neutral strategies, we show that incorporating industry level information in the forecasting model generally yields the lowest beta values for the portfolio, particularly at the short and intermediate forecast horizons. This suggests that beta forecasts obtained via models that employ past industry can help achieve beta neutral portfolios for diversified investors more effectively than the other forecasting models that incorporate financial, macroeconomic, sentiment, and climate-based factors. Overall, while our results show that the multivariate framework proposed in our analysis can improve the forecast accuracy for realized covariances, they also suggest that the predictor set utilized in forecasting models must be aligned with the target investment horizon.

The findings have important implications for portfolio allocation strategies. Considering the evidence from the forecasting literature that low-frequency volatility models generally underperform their high-frequency counterparts (Koopman et al., 2005; Horpestad et al., 2019; Lyócsa et al., 2021), the MHAR framework coupled with DRD decomposition proposed in this paper can be utilized in a high frequency setting in order to implement diversification strategies for various investment horizons. Furthermore, given that the realized covariance forecasts can be used to generate forecasts for asset betas, the proposed framework can be used to forecast factor betas that are an indispensable part of factor strategies that aim to exploit persistent

drivers of returns based on their factor exposures. In future work, it will be interesting to examine the performance of the proposed multivariate model on a diversified portfolio, covering various asset classes such as bonds, equities, commodities and cryptocurrencies. It will also be interesting to examine whether the forecasted betas obtained via the MHAR models can be used to predict subsequent return patterns.

References

Ang, A., & Chen, J. (2001). Asymmetric correlations of equity portfolios. Journal of Financial Economics, 63(3), 637– 654.

Asgharian, H., Hou, A. J., & Javed, F. (2013). The importance of the macroeconomic variables in forecasting stock return variance: A GARCH-MIDAS approach. Journal of Forecasting, 32, 600–612.

Ayub, U., Shah, S. Z. A., & Abbas, Q. (2015). Robust analysis for downside risk in portfolio management for a volatile stock market. Economic Modelling, 44, 86–96.

Baker, M., & Wurgler, J. (2006). Investor sentiment and the cross-section of stock returns. The Journal of Finance, 61(4), 1645-1680.

Baker, S., Bloom, N., Davis, S., 2016. Measuring economic policy uncertainty. The Quarterly Journal of Economics, 131, 1593–1636.

Bee, M., Dupuis, D. J., & Trapin, L. (2016). Realizing the extremes: Estimation of tail-risk measures from a high-frequency perspective. Journal of Empirical Finance, 36, 86–99.

Bekaert, G., Engstrom, E., Xu. N. (2022). The Time Variation in Risk Appetite and Uncertainty. Management Science, Vol. 68, No. 6, 3975–4004.

Bollerslev, T., Patton, A. J., & Quaedvlieg, R. (2018). Modeling and forecasting (un) reliable realized covariances for more reliable financial decisions. Journal of Econometrics, 207(1), 71-91.

Brown, D. B., and Smith, J. E. (2011). Dynamic portfolio optimization with transaction costs: Heuristics and dual bounds. Management Science, 57(10), 1752-1770.

Bolton, P. and Kacperczyk, M. 2021. Do investors care about carbon risk? Journal of Financial Economics 142, 517–549.

Callot, L. A., Kock, A. B., and Medeiros, M. C. (2017). Modeling and forecasting large realized covariance matrices and portfolio choice. Journal of Applied Econometrics, 32(1), 140-158.

Cederburg, S., O'Doherty, M. S., Wang, F., & Yan, X. (2020). On the performance of volatility-managed portfolios. Journal of Financial Economics, 138, 95–117.

Cepni, O., Demirer, R., Rognone, L. 2022. Hedging Climate Risks with Green Assets. Economics Letters 212, 110312.

Chen, E. T. J., and Clements, A. (2007). S&P 500 implied volatility and monetary policy announcements. Finance Research Letters, 4(4), 227-232.

Choi, D., Gao, Z., & Jiang, W. (2020). Attention to global warming. The Review of Financial Studies, 33(3), 1112-1145.

Christensen, K., Siggaard, M., & Veliyev, B. (2021). A machine learning approach to volatility forecasting. Available at SSRN.

Clements, A., & Liao, Y. (2017). Forecasting the variance of stock index returns using jumps and cojumps. International Journal of Forecasting, 33, 729–742.

Clements, A., & Preve, D. P. (2021). A practical guide to harnessing the har volatility model. Journal of Banking & Finance, 133, 106285.

Corsi, F. (2009). A simple approximate long-memory model of realized volatility. Journal of Financial Econometrics, 7(2), 174-196.

DeMiguel, V., Nogales, F. J., and Uppal, R. (2014). Stock return serial dependence and out-of-sample portfolio performance.The Review of Financial Studies, 27(4), 1031-1073.

Demirer, R., Gupta, R. and Pierdzioch, C. (2020) Forecasting Realized Stock-Market Volatility: Do Industry Returns Have Predictive Value? (December 8, 2020). Available at SSRN: https://ssrn.com/abstract=3744537

Engle, R.F., and Rangel, J.G. (2008). The Spline-GARCH Model for Low-Frequency Volatility and Its Global Macroeconomic Causes. Review of Financial Studies 21(3), 1187-1222.

Engle, R.F., Ghysels, E., and Sohn, B. (2013). Stock Market Volatility and Macroeconomic Fundamentals. The Review of Economics and Statistics 95(3), 776-797.

Faccini, R., Matin, R. and Skiadopoulos, G. 2021. Are Climate Change Risks Priced in the US Stock Market? Danmarks Nationalbank Working Paper, No. 169, February.

Graham, J. R., & Harvey, C. R. (1996). Market timing ability and volatility implied in investment newsletters' asset allocation recommendations. Journal of Financial Economics, 42, 397–421.

Gupta, R.,Hammoudeh, S., Modise,M. P.,&Nguyen, D. K. (2014). Can economic uncertainty, financial stress and consumer sentiments predict U.S. equity premium? Journal of International Financial Markets, Institutions and Money, 33, 367–378.

Golosnoy, V., & Gribisch, B. (2022). Modeling and forecasting realized portfolio weights. Journal of Banking & Finance, 138, 106404.

Hong, H., Torous, W., Valkanov, R. (2007). Do industries lead stock markets? Journal of Financial Economics, 83: 367-396.

Hong, H., Torous, W., Valkanov, R., (2014). Note on "Do industries lead stock markets?". http://rady.ucsd.edu/docs/faculty/valkanov/Note_10282014.pdf.

Horpestad, J. B., Lyócsa, Š., Molnár, P., & Olsen, T. B. (2019). Asymmetric volatility in equity markets around the world. The North American Journal of Economics and Finance, 48, 540-554.

Ji, Q., Zhang, D., & Zhao, Y. (2022). Intra-day co-movements of crude oil futures: China and the international benchmarks. Annals of Operations Research 313, 77–103.

Jin, Z., Guo, K., Sun, Y., Lai, L., & Liao, Z. (2020). The industrial asymmetry of the stock price prediction with investor sentiment: Based on the comparison of predictive effects with SVR. Journal of Forecasting, 39, 1166–1178.

Kempf, A., & Memmel, C. (2006). Estimating the global minimum variance portfolio. Schmalenbach Business Review, 58(4), 332-348.

Li, X., &Wei, Y. (2018). The dependence and risk spillover between crude oil market and China stock market: New evidence from a variational mode decomposition-based copula method. Energy Economics, 74, 565–581.

Li, X., Wei, Y., Chen, X., Ma, F., Liang, C., & Chen, W. (2022). Which uncertainty is powerful to forecast crude oil market volatility? New evidence. International Journal of Finance and Economics 27 (4), 4279-4297.

Liang, C., Tang, L., Li, Y., & Wei, Y. (2020). Which sentiment index is more informative to forecast stock market volatility? Evidence from China. International Review of Financial

Analysis, 71, 101552.

Koopman, S. J., Jungbacker, B., & Hol, E. (2005). Forecasting daily variability of the S&P 100 stock index using historical, realised and implied volatility measurements. Journal of Empirical Finance, 12(3), 445-475.

Lyócsa, Š., Molnár, P., & Výrost, T. (2021). Stock market volatility forecasting: Do we need high-frequency data?. International Journal of Forecasting, 37(3), 1092-1110.

Nonejad, N. (2017). Forecasting aggregate stock market volatility using financial and macroeconomic predictors: Which models forecast best, when and why? Journal of Empirical Finance, 42, 131–154.

Paye, B. S. (2012). 'Déjà vol': Predictive regressions for aggregate stock market volatility using macroeconomic variables. Journal of Financial Economics, 106, 527–546.

Poon, S-H, and Granger, C. W. J. (2003). Forecasting Volatility in Financial Markets: A Review. Journal of Economic Literature, 41(2), 478-539.

Rangel, J.G., and Engle, R.F. (2011). The Factor-Spline-GARCH Model for High and Low Frequency Correlations. Journal of Business & Economic Statistics 30(1), 109-124.

Rapach, D.E., Strauss, J.K., and Wohar, M.E. (2008). Forecasting stock return volatility in the presence of structural breaks, in Forecasting in the Presence of Structural Breaks and Model Uncertainty, in David E. Rapach and Mark E. Wohar (Eds.), Vol. 3 of Frontiers of Economics and Globalization, Bingley, United Kingdom: Emerald (May 2008), 381-416.

Singh, A. (2016). On the linkages between India VIX and US financial stress index. Theoretical Economics Letters, 06, 68–74.

Sum, V. (2014). Dynamic effects of financial stress on the U.S. real estate market performance. Journal of Economics and Business, 75, 80–92.

Symitsi, E., Symeonidis, L., Kourtis, A., & Markellos, R. (2018). Covariance forecasting in equity markets. Journal of Banking & Finance, 96, 153-168.

Tetlock, P. C. (2007). Giving content to investor sentiment: The role of media in the stock market. The Journal of Finance, 62(3), 1139-1168.

Vahamaa, S., and Aijo, J. (2011). The Fed's policy decisions and implied volatility. The Journal of Futures Markets, 31(10), 995-1009.

Wang, X.,Wu, C., & Xu,W. (2015). Volatility forecasting: The role of lunch-break returns, overnight returns, trading volume and leverage effects. International Journal of Forecasting, 31, 609–619.

Wang, J., Lu, X., He, F.,& Ma, F. (2020). Which popular predictor is more useful to forecast international stock markets during the coronavirus pandemic: VIX vs EPU? International Review of Financial Analysis, 72, 101596.

Wang, Y., Ma, F., Wei, Y., & Wu, C. (2016). Forecasting realized volatility in a changing world: A dynamic model averaging approach. Journal of Banking & Finance, 64, 136-149.

Zhu, K., & Ling, S. (2015). Model-based pricing for financial derivatives. Journal of Econometrics, 187, 447–457.

Zhang, H., He, Q., Jacobsen, B., & Jiang, F. (2020). Forecasting stock returns with model uncertainty and parameter instability. Journal of Applied Econometrics, 35(5), 629-644.

Table 1: Descriptive statistics of realized volatility estimates.

Stock indices	Mean	Std. dev	Skewness	Kurtosis	JB (p-value)	Q(5)	Q(10)	Q(20)	ADF
SPX Index	0.44	0.78	8.62	121.87	1339421.58***	2880.35***	3842.83***	4862.67***	-15.29***
INFT Index	0.68	1.15	10.52	181.20	2989095.12***	1482.14***	1866.40***	2252.28***	-18.82***
HLTH Index	0.53	0.90	10.40	177.14	2855186.38***	1687.04***	2231.10***	2762.59***	-18.56***
COND Index	0.58	0.96	7.67	99.38	884256.94***	2780.93***	3885.06***	5137.49***	-15.25***
TELS Index	0.63	0.76	6.79	78.15	541438.55***	1945.22***	2559.83***	3222.38***	-17.91***
FINL Index	0.73	1.16	8.19	110.55	1098734.67***	3369.69***	4808.36***	6665.98***	-14.94***
INDU Index	0.58	0.92	6.90	77.60	534282.21***	3570.61***	5098.20***	7110.43***	-14.27***
CONS Index	0.34	0.54	14.56	373.65	12832346.99***	1455.39***	1877.48***	2221.07***	-18.66***
MATR Index	0.72	1.00	6.01	59.45	309240.60***	4194.02***	6188.14***	8723.26***	-13.42***
UTIL Index	0.56	0.65	11.74	249.07	5672127.34***	1159.20***	1430.75***	1665.55***	-20.27***
ENRS Index	1.07	1.36	5.57	53.75	250610.33***	3786.82***	5378.94***	7247.37***	-14.25***

Note: This table reports the descriptive statistics for daily realized volatility estimates obtained from intraday returns for the S&P 500 index and its eleven industry sub-indices, namely: Information technology (INFT), Health care (HLTH), Consumer discretionary (COND), Communication services (TELS), Financials (FINL), Industrials (INDU), Consumer staples (CONS), Materials (MATR), Utilities (UTIL), Energy (ENRS). We report the mean, standard deviation, skewness and kurtosis of the RV series in addition to the JB statistic, the Ljung-box test statistic and the ADF test results. *** denotes the significance at the 1% level.

Table 2: Forecast accuracy measures based on RMSE and Stein loss functions.

	RMSE	MCS_{TR}	MCS_{TSQ}	Stein loss	MCS_t	MCS_sq
			h=1			
HARDRD	12.4400	0.0039	0.0073	35.5424	0.0030	0.0130
HARDRD-Finance	12.4618	0.0039	0.0073	37.3110	0.0016	0.0048
LASSO-HARDRD-Finance	12.2316	0.0275	0.0275	32.6745	0.0404	0.0432
HARDRD-Macro	12.6423	0.0039	0.0073	37.9188	0.0030	0.0130
LASSO-HARDRD-Macro	12.4643	0.0039	0.0073	34.3109	0.0199	0.0432
HARDRD-Sentiment	12.6450	0.0039	0.0073	37.2412	0.0016	0.0066
LASSO-HARDRD-Sentiment	12.3233	0.0039	0.0073	33.4996	0.0199	0.0432
HARDRD-climate	12.4483	0.0039	0.0073	35.6627	0.0030	0.0130
LASSO-HARDRD-climate	**12.2255**	**1.0000****	**1.0000****	32.5187	0.0404	0.0432
HARDRD-volatility	12.4872	0.0039	0.0059	37.9598	0.0016	0.0052
LASSO-HARDRD-volatility	12.2665	0.0039	0.0073	**32.1973**	**1.0000****	**1.0000****
HARDRD-All	12.6324	0.0039	0.0059	43.3760	0.0030	0.0130
LASSO-HARDRD-All	12.4231	0.0039	0.0073	33.6289	0.0404	0.0432
HARDRD-factor	12.5276	0.0039	0.0073	37.0354	0.0025	0.0079
LASSO-HARDRD-factor	12.3505	0.0039	0.0073	34.4393	0.0030	0.0150
			h=5			
HARDRD	11.8425	0.1817*	0.1243*	32.4971	0.0098	0.0381
HARDRD-Finance	11.7925	0.3415**	0.2268*	34.1467	0.0411	0.0649
LASSO-HARDRD-Finance	11.7683	0.3415**	0.2268*	31.5972	0.1031*	0.0844
HARDRD-Macro	11.8440	0.2804**	0.1557*	32.1067	0.0411	0.0720
LASSO-HARDRD-Macro	11.8004	0.3415**	0.2132*	32.0453	0.0411	0.0649
HARDRD-Sentiment	11.7763	0.3415**	0.2917**	31.5193	0.1031*	0.0909
LASSO-HARDRD-Sentiment	**11.7072**	**1.0000****	**1.0000****	31.8302	0.0997	0.0789
HARDRD-climate	11.8287	0.3415**	0.2132*	31.3486	0.1031*	0.0909
LASSO-HARDRD-climate	11.7368	0.5335**	0.5335**	30.9975	0.1379*	0.0918
HARDRD-volatility	11.8071	0.3415**	0.2268*	32.4475	0.0997	0.0789
LASSO-HARDRD-volatility	11.7562	0.3449**	0.3497**	30.1879	0.4802**	0.4802**
HARDRD-All	12.0229	0.1817*	0.0857	33.1018	0.0411	0.0538
LASSO-HARDRD-All	11.8082	0.3415**	0.2268*	31.3806	0.1102*	0.0909
HARDRD-factor	11.8905	0.3415**	0.1685*	32.0887	0.1031*	0.0909
LASSO-HARDRD-factor	11.8277	0.3415**	0.2132*	**29.5949**	**1.0000****	**1.0000****
			h=22			
HARDRD	11.2618	0.0456	0.0290	27.9231	0.0072	0.0208
HARDRD-Finance	11.2929	0.0456	0.0250	27.1798	0.0072	0.0542
LASSO-HARDRD-Finance	11.2007	0.0456	0.0305	27.5035	0.0072	0.0375
HARDRD-Macro	11.4515	0.0194	0.0212	27.2822	0.0072	0.0611
LASSO-HARDRD-Macro	11.3725	0.0194	0.0212	26.8927	0.0072	0.0611
HARDRD-Sentiment	11.4138	0.0182	0.0154	26.9028	0.0072	0.0611
LASSO-HARDRD-Sentiment	11.3385	0.0194	0.0212	26.4335	0.0731	0.1271*
HARDRD-climate	11.2612	0.0456	0.0305	26.6368	0.0731	0.1271*
LASSO-HARDRD-climate	**11.1510**	**1.0000****	**1.0000****	27.0505	0.0119	0.0782
HARDRD-volatility	11.3614	0.0456	0.0250	26.9981	0.0731	0.1271*
LASSO-HARDRD-volatility	11.2722	0.0456	0.0305	**25.7345**	**1.0000****	**1.0000****
HARDRD-All	11.4502	0.0456	0.0250	26.8817	0.0731	0.1271*
LASSO-HARDRD-All	11.4432	0.0194	0.0212	26.7395	0.0119	0.0861
HARDRD-factor	11.3392	0.0194	0.0196	25.9678	0.4591**	0.4591**
LASSO-HARDRD-factor	11.2517	0.0456	0.0305	26.1589	0.4131**	0.3977**

Note: This table presents the pairwise comparisons of the out-of-sample forecasts from the benchmark HARDRD model against the extended model variations based on RMSE and Stein loss functions. The panels report the findings for the one-step, five-step and 22-step forecasts of realized covariance. The lower values of RMSE and Stein loss suggest the higher precision for the realized covariance forecasts. We also report the MCS results to test the significance of differences in forecast precision across the models. The columns denoted by "MCS_{TR}" and "MCS_{TSQ}" are the MCS test results given the range statistics, T_R and the semi-quadratic statistics T_{SQ} respectively. We denote the models that are included in the MCS at 10% and 25% confidence levels with * and ** respectively. Lower loss function values and ** indicate better forecasting performance.

Table 3: Economic evaluation of industry portfolios

				c =0%	c =1%	c =2%	c =0%	c =1%	c =2%		γ=1			γ=10	
	GMVP	Turnover	CO	Sharpe1	Sharpe2	Sharpe3	return1	return2	return3	EV1_1	EV1_2	EV1_3	EV2_1	EV2_2	EV2_3
								h=1							
HARDRD	0.1813	0.6993	0.5863	0.0472	0.0363	0.0254	7.6449	5.8828	4.1206	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARDRD-Finance	0.1827	0.7019	0.5877	0.0477	0.0368	0.0259	7.7314	5.9626	4.1939	0.1241	0.1147	0.1053	0.1141	0.1052	0.0962
LASSO-HARDRD-Finance	0.1738	0.6253	0.5563	0.0471	0.0374	0.0276	7.6151	6.0394	4.4637	-0.0399	0.2304	0.5006	-0.0084	0.2632	0.5348
HARDRD-Macro	0.1954	0.4526	0.5627	0.0468	0.0397	0.0326	7.5572	6.4168	5.2763	-0.1202	0.7806	1.6815	-0.0556	0.8480	1.7520
LASSO-HARDRD-Macro	0.1866	0.5772	0.5424	0.0437	0.0347	0.0257	7.0381	5.5834	4.1288	-0.8677	-0.4223	0.0232	-0.7615	-0.3161	0.1294
HARDRD-Sentiment	0.1904	0.4945	0.5821	0.0475	0.0398	0.0321	7.6788	6.4328	5.1867	0.0561	0.8040	1.5520	0.1223	0.8737	1.6255
LASSO-HARDRD-Sentiment	0.1783	0.5933	0.5553	0.0469	0.0376	0.0284	7.5732	6.0781	4.5831	-0.0997	0.2874	0.6745	-0.0598	0.3291	0.7181
HARDRD-climate	0.1818	0.7053	0.5878	0.0457	0.0347	0.0237	7.3880	5.6107	3.8334	-0.3691	-0.3909	-0.4126	-0.3403	-0.3612	-0.3821
LASSO-HARDRD-climate	0.1737	0.6225	0.5551	0.0468	0.0371	0.0273	7.5490	5.9803	4.4116	-0.1325	0.1479	0.4283	-0.0713	0.2101	0.4916
HARDRD-volatility	0.1860	0.3676	0.5659	0.0467	0.0410	0.0352	7.5466	6.6202	5.6938	-0.1349	1.0759	2.2868	-0.0642	1.1499	2.3645
LASSO-HARDRD-volatility	0.1813	0.4574	0.5285	0.0441	0.0369	0.0298	7.1105	5.9577	4.8050	-0.7674	0.1155	0.9984	-0.7043	0.1807	1.0661
HARDRD-All	0.1875	0.4104	0.5808	0.0430	0.0367	0.0305	7.1078	6.0736	5.0394	-0.8052	0.2498	1.3049	-1.0627	-0.0015	1.0602
LASSO-HARDRD-All	0.1857	0.4904	0.5283	0.0458	0.0381	0.0304	7.3722	6.1363	4.9004	-0.3857	0.3767	1.1392	-0.2984	0.4659	1.2306
HARDRD-factor	0.1885	0.3771	0.5635	0.0485	0.0427	0.0368	7.8549	6.9045	5.9542	0.3079	1.4840	2.6602	0.3440	1.5223	2.7013
LASSO-HARDRD-factor	0.1815	0.5964	0.5507	0.0454	0.0360	0.0267	7.3174	5.8146	4.3117	-0.4678	-0.0921	0.2837	-0.4054	-0.0289	0.3477
								h=5							
HARDRD	0.2066	0.1811	0.6006	0.0926	0.0863	0.0800	6.7288	6.2724	5.8160	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARDRD-Finance	0.2023	0.1755	0.6038	0.0986	0.0926	0.0865	7.1754	6.7332	6.2910	0.6467	0.6673	0.6879	0.6452	0.6662	0.6871
LASSO-HARDRD-Finance	0.2032	0.1687	0.5936	0.0956	0.0898	0.0839	6.9059	6.4807	6.0555	0.2586	0.3040	0.3493	0.2789	0.3254	0.3720
HARDRD-Macro	0.2082	0.1340	0.5956	0.0916	0.0869	0.0822	6.5741	6.2364	5.8987	-0.2205	-0.0486	0.1234	-0.1870	-0.0145	0.1580
LASSO-HARDRD-Macro	0.2025	0.1620	0.5869	0.0963	0.0906	0.0850	6.9780	6.5697	6.1614	0.3626	0.4323	0.5020	0.3780	0.4482	0.5184
HARDRD-Sentiment	0.2070	0.1570	0.6027	0.0981	0.0927	0.0873	7.1980	6.8024	6.4069	0.6775	0.7657	0.8539	0.6580	0.7464	0.8349
LASSO-HARDRD-Sentiment	0.2031	0.1744	0.5961	0.0987	0.0927	0.0867	7.2162	6.7767	6.3373	0.7042	0.7290	0.7537	0.6882	0.7145	0.7409
HARDRD-climate	0.2094	0.1919	0.6006	0.0956	0.0890	0.0823	6.9651	6.4815	5.9979	0.3412	0.3018	0.2624	0.3316	0.2921	0.2526
LASSO-HARDRD-climate	0.2031	0.1328	0.5858	0.0932	0.0886	0.0840	6.7371	6.4024	6.0678	0.0145	0.1909	0.3674	0.0381	0.2156	0.3930
HARDRD-volatility	0.2056	0.1663	0.5986	0.0939	0.0881	0.0823	6.7988	6.3797	5.9607	0.1030	0.1570	0.2111	0.1179	0.1720	0.2260
LASSO-HARDRD-volatility	0.2044	0.1384	0.5822	0.0956	0.0908	0.0860	6.9226	6.5738	6.2250	0.2825	0.4385	0.5945	0.2996	0.4566	0.6136
HARDRD-All	0.2131	0.2371	0.6081	0.1110	0.1028	0.0946	8.0798	7.4824	6.8849	1.9551	1.7508	1.5466	1.9392	1.7349	1.5305

LASSO-HARDRD-All	0.2051	0.2082	0.5931	0.1155	0.1083	0.1010	8.3629	7.8383	7.3137	2.3679	2.2691	2.1702	2.3779	2.2786	2.1792
HARDRD-factor	0.2094	0.2469	0.5987	0.1001	0.0915	0.0829	7.2339	6.6117	5.9896	0.7351	0.4949	0.2547	0.7687	0.5270	0.2853
LASSO-HARDRD-factor	0.2076	0.1528	0.5830	0.1015	0.0962	0.0908	7.2986	6.9135	6.5284	0.8298	0.9331	1.0365	0.8724	0.9763	1.0802
								h=22							
HARDRD	0.2292	0.1162	0.6134	0.2266	0.2170	0.2075	6.9098	6.6170	6.3242	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARDRD-Finance	0.2223	0.1239	0.6342	0.1998	0.1898	0.1797	6.1894	5.8770	5.5647	-1.0448	-1.0730	-1.1013	-1.0565	-1.0849	-1.1133
LASSO-HARDRD-Finance	0.2258	0.1357	0.6069	0.2128	0.2016	0.1905	6.5343	6.1924	5.8505	-0.5444	-0.6156	-0.6867	-0.5491	-0.6210	-0.6929
HARDRD-Macro	0.2383	0.1038	0.6280	0.2123	0.2039	0.1954	6.5518	6.2901	6.0285	-0.5188	-0.4737	-0.4285	-0.5209	-0.4760	-0.4311
LASSO-HARDRD-Macro	0.2368	0.1296	0.6099	0.2049	0.1942	0.1836	6.3362	6.0096	5.6829	-0.8311	-0.8802	-0.9293	-0.8337	-0.8836	-0.9336
HARDRD-Sentiment	0.2335	0.1214	0.6162	0.2057	0.1959	0.1862	6.4405	6.1346	5.8287	-0.6818	-0.7007	-0.7197	-0.7003	-0.7197	-0.7391
LASSO-HARDRD-Sentiment	0.2329	0.1405	0.6052	0.2335	0.2218	0.2101	7.0671	6.7130	6.3589	0.2285	0.1397	0.0510	0.2361	0.1467	0.0573
HARDRD-climate	0.2338	0.1237	0.6098	0.2367	0.2265	0.2162	7.1929	6.8811	6.5693	0.4106	0.3830	0.3554	0.4152	0.3874	0.3595
LASSO-HARDRD-climate	0.2239	0.1242	0.6049	0.2189	0.2088	0.1986	6.7225	6.4095	6.0965	-0.2714	-0.3007	-0.3301	-0.2723	-0.3024	-0.3324
HARDRD-volatility	0.2374	0.1185	0.6239	0.2383	0.2285	0.2187	7.2330	6.9343	6.6356	0.4687	0.4601	0.4515	0.4744	0.4651	0.4559
LASSO-HARDRD-volatility	0.2315	0.1386	0.6007	0.2377	0.2260	0.2144	7.1527	6.8035	6.4543	0.3520	0.2702	0.1883	0.3538	0.2709	0.1879
HARDRD-All	0.2292	0.1565	0.6403	0.2240	0.2116	0.1992	7.0953	6.7009	6.3065	0.2698	0.1225	-0.0248	0.2806	0.1315	-0.0176
LASSO-HARDRD-All	0.2296	0.1446	0.6288	0.2131	0.2015	0.1900	6.7377	6.3732	6.0087	-0.2493	-0.3533	-0.4572	-0.2493	-0.3547	-0.4601
HARDRD-factor	0.2228	0.1085	0.6394	0.2035	0.1947	0.1859	6.3299	6.0566	5.7832	-0.8412	-0.8130	-0.7848	-0.8525	-0.8248	-0.7971
LASSO-HARDRD-factor	0.2282	0.1322	0.6046	0.2089	0.1981	0.1872	6.4210	6.0879	5.7547	-0.7077	-0.7661	-0.8246	-0.7050	-0.7637	-0.8225

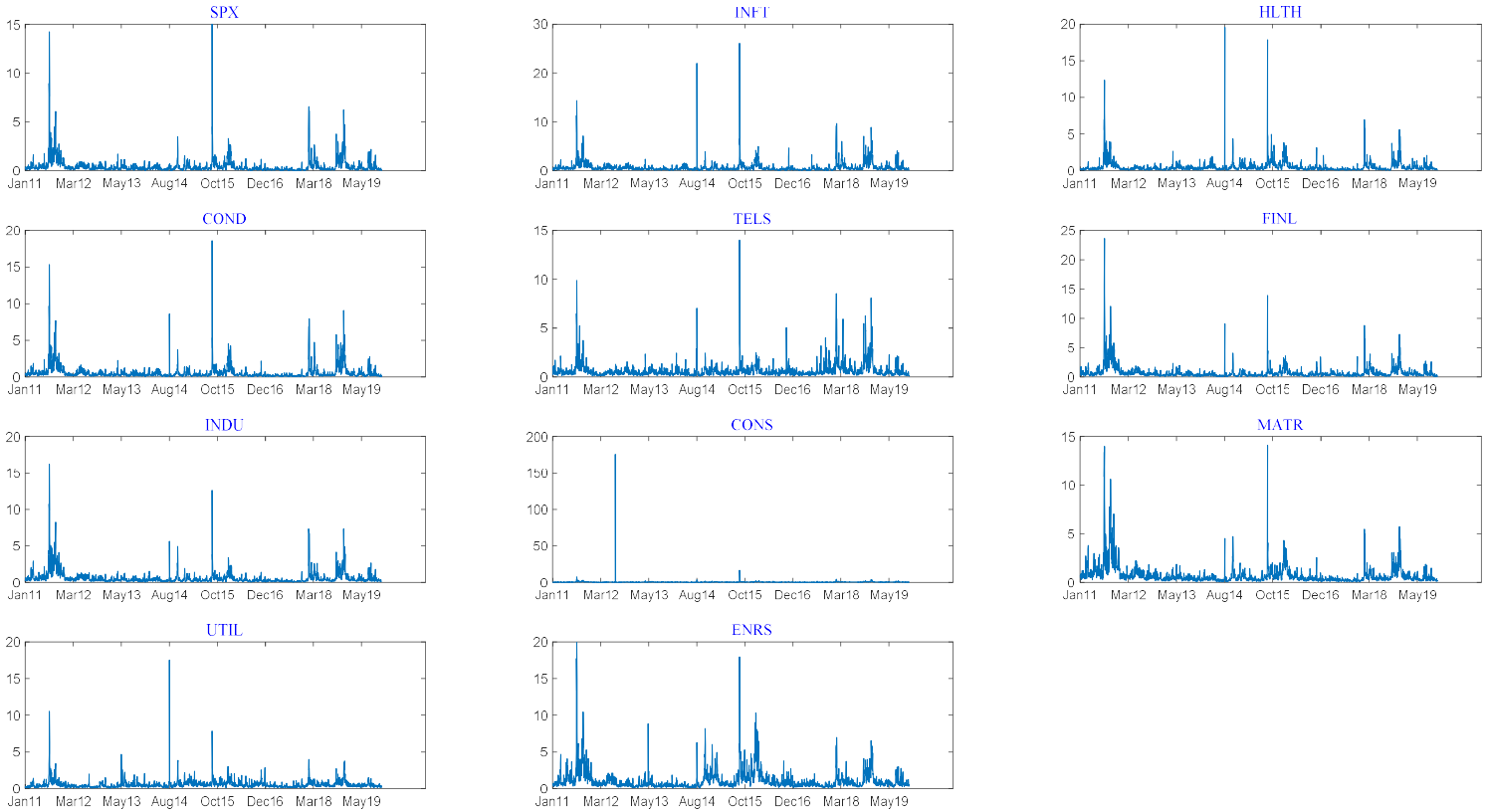
Note: The table reports the economic evaluation results for the constructed industry portfolios obtained from competing forecasting models for h-step ahead forecasts. We report the volatility forecast for the Global Minimum Variance Portfolio (GMVP), the average turnover ratio, the average concentration ration (CO), annualized average out-of-sample return and the Sharpe Ratios (SR) based on three alternative transaction costs, c= 0, 1%, 2%. We also report the Economic Value (EV) of the constructed portfolios over the benchmark HARDRD model obtained using two risk aversion rates, γ=1 and γ=10. Lower values for the GMVP, the average turnover ratio, the average concentration ration (CO) indicate improved portfolio performance.

Table 4: Beta-neutral portfolios

	h=1		h=5		h=22	
	Beta_neutral	t_statistics	Beta_neutral	t_statistics	Beta_neutral	t_statistics
HARDRD	0.0839***	5.0226	0.0547***	2.9111	0.0883***	5.0056
HARDRD-Finance	0.0793***	4.7372	0.0633***	3.3978	0.0829***	4.8244
LASSO-HARDRD-Finance	0.0463***	2.7150	0.0485***	2.5236	0.0755***	4.2874
HARDRD-Macro	0.0598***	3.7297	0.0435***	2.4327	0.0843***	4.7987
LASSO-HARDRD-Macro	0.0346**	2.1370	0.0366**	1.9909	0.0738***	4.1900
HARDRD-Sentiment	0.0489***	2.8152	0.0703***	3.6670	0.0815***	4.6305
LASSO-HARDRD-Sentiment	0.0368**	2.0702	0.0527***	2.6963	0.0718***	3.9563
HARDRD-climate	0.0871***	5.2997	0.0532***	2.8064	0.0796***	4.4026
LASSO-HARDRD-climate	0.0466***	2.7898	0.0371*	1.9528	0.0668***	3.7133
HARDRD-volatility	0.0587***	3.4350	0.0545***	2.9650	0.0840***	4.7892
LASSO-HARDRD-volatility	0.0184	1.0958	0.0291	1.5399	0.0676***	3.6974
HARDRD-All	0.0908***	5.4512	0.0765***	3.9779	0.0900***	5.1619
LASSO-HARDRD-All	0.0048	0.2897	0.0520***	2.7993	0.0821***	4.7204
HARDRD-factor	0.0380**	2.2052	0.0522***	2.8696	0.1002***	5.7281
LASSO-HARDRD-factor	0.0326*	1.8950	0.0379**	2.0167	0.0775***	4.2372

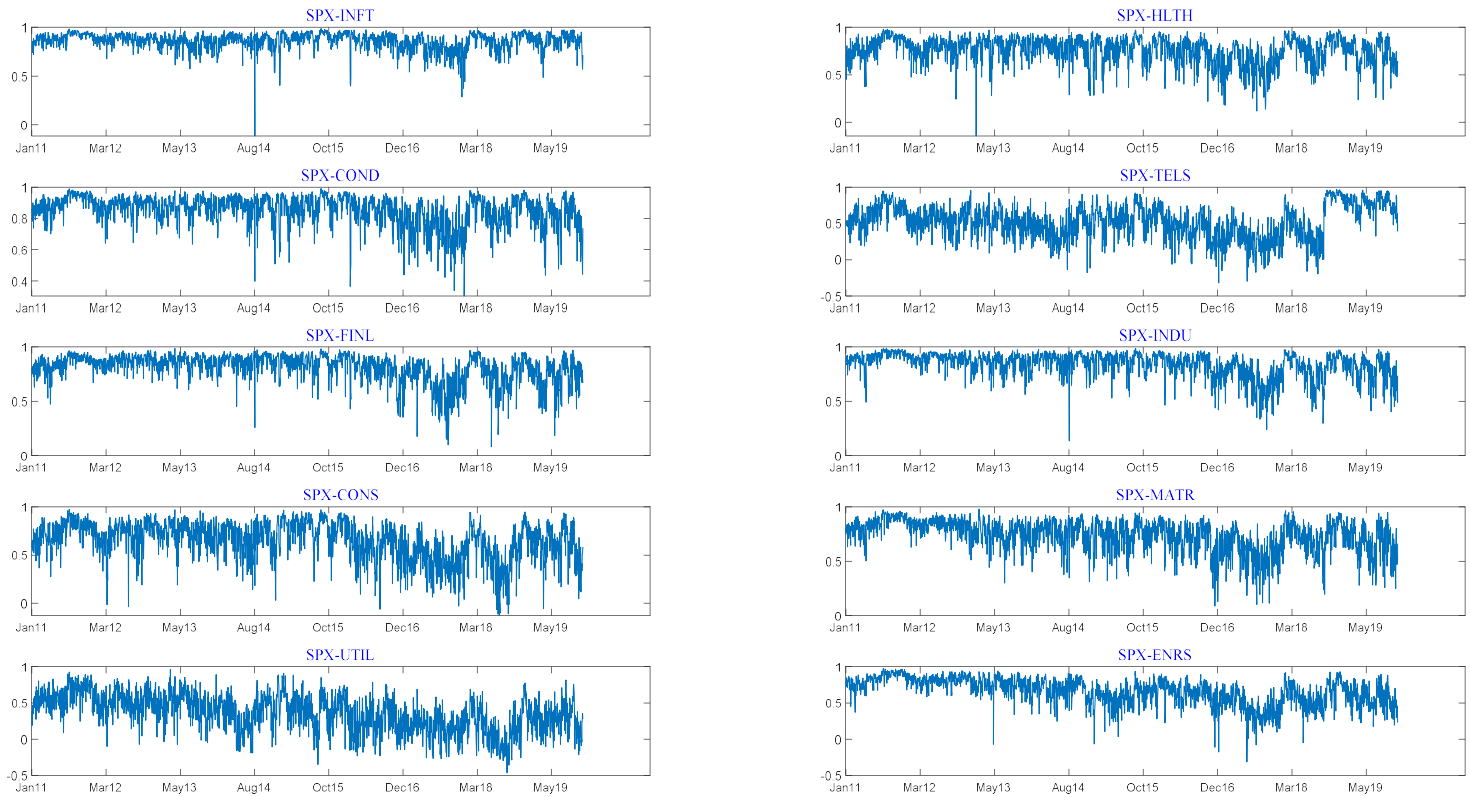
Note: This table presents the actual betas for the beta-neutral portfolios constructed using the industry beta forecasts ($\hat{\beta}$) obtained from competing forecasting models for h-step ahead forecasts. For each forecast approach, we sort the 10 industry indices into high and low beta groups based on their respective current beta forecasts according to Equation 31. We then construct the beta-neutral portfolios ex ante by solving the equation $\upsilon_{j,t}\hat{\beta}_{long\ j,t} - \hat{\beta}_{short,j,t} = 0$ and applying the resulting weight $\upsilon_{j,t}$ to the long leg of the portfolios. We also compute the t-statistics according to the robust Newey and West (1987) standard errors with 6 lags, in order to test whether the actual beta of the portfolio is significantly different from 0. An estimated portfolio beta value close to 0 and insignificant t-statistic imply that the portfolios is market-neutral.

Figure 1: Realized volatilities of SP500 index and industry indices.



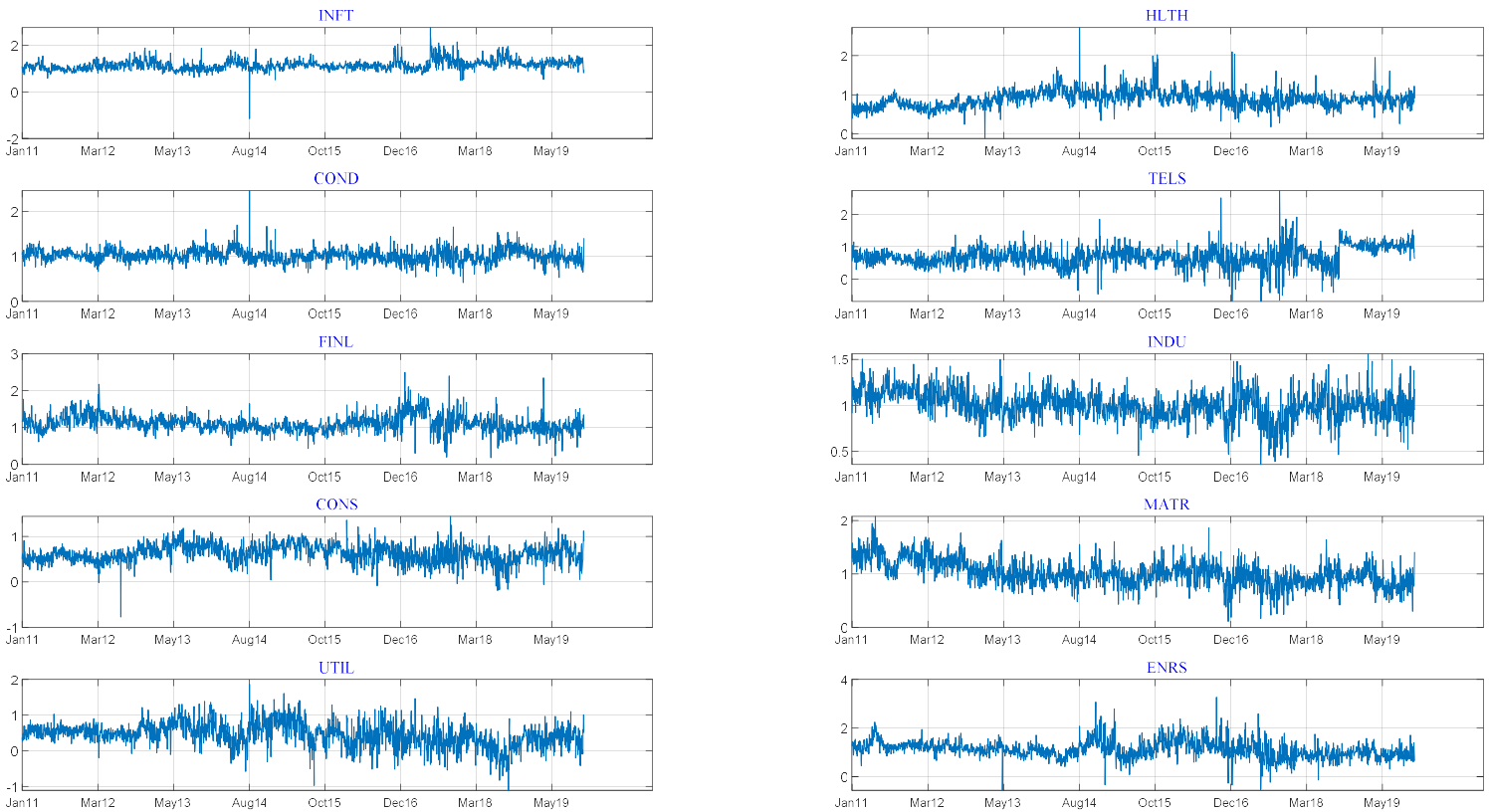
Note: The figure presents the daily realized volatility estimates obtained from intraday returns. Information technology (INFT), Health care (HLTH), Consumer discretionary (COND), Communication services (TELS), Financials (FINL), Industrials (INDU), Consumer staples (CONS), Materials (MATR), Utilities (UTIL), Energy (ENRS).

Figure 2. Realized correlations of industry indices with the market index.



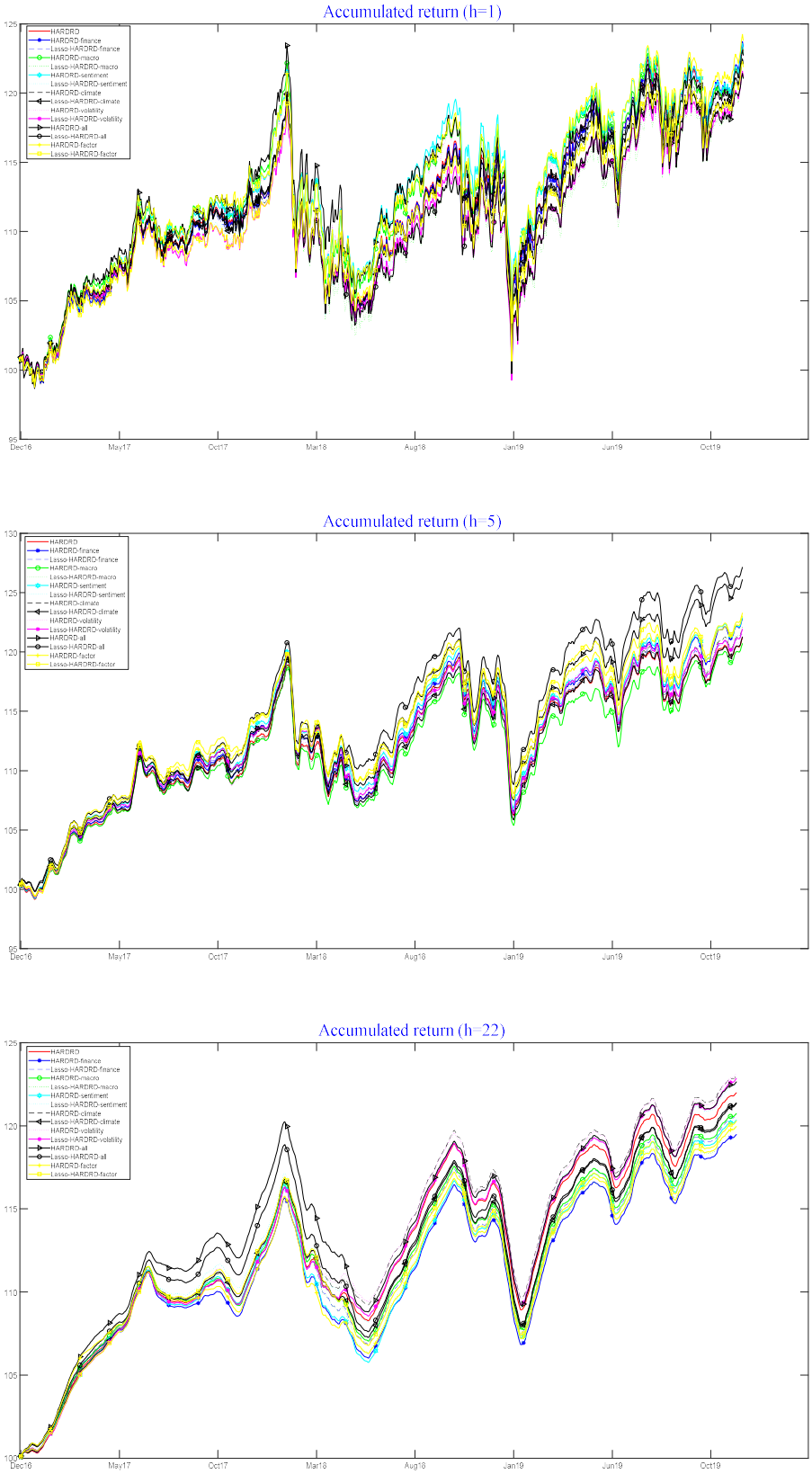
Note: The figure presents the daily realized correlation estimates between the S&P 500 index and each industry obtained from intraday returns. Information technology (INFT), Health care (HLTH), Consumer discretionary (COND), Communication services (TELS), Financials (FINL), Industrials (INDU), Consumer staples (CONS), Materials (MATR), Utilities (UTIL), Energy (ENRS).

Figure 3. Realized industry betas.



Note: The figure presents the daily realized beta estimates for each industry obtained from intraday returns. Information technology (INFT), Health care (HLTH), Consumer discretionary (COND), Communication services (TELS), Financials (FINL), Industrials (INDU), Consumer staples (CONS), Materials (MATR), Utilities (UTIL), Energy (ENRS).

Figure 4: Cumulative portfolio returns for different MHAR models (short out-of-sample).



Appendix

Table A1: Description of exogenous predictors

Name	**Name**
Financial market variables	Cboe Oil ETF VIX Index
	Cboe Gold ETF VIX Index
	Chicago Board Options Exchange Volatility Index
	ICE BofA MOVE Index
	The CBOE EuroCurrency Volatility
	J.P. Morgan G7 Volatility Index (VXY)
	Risk aversion index
	MIAX/T3 Volatility Index
	GS US Financial Conditions Index
Macroeconomic variables	Morgan Stanley US Shadow Short Rate
	Aruoba Diebold Scotti Business Conditions Index »
	US 10Y Breakeven inflation rate
	GS Economic Surprise Index
	US Economic Policy Uncertainty
	BDI Baltic Exchange Dry Index
Sentiment variables	Daily News Sentiment Index
	Citi FX Forecast Uncertainty - USD vs Emerging Markets Currencies
	US Infectious Disease Equity Market Volatility Tracker (EMV) Daily
	Nasdaq US Insider Sentiment Index
	Bloomberg Closing ARMs for New York Stock Exchange
	Bloomberg Closing ARMs for Nasdaq Composite
	Bloomberg Closing ARMs for American Stock Exchange
	Bloomberg Total Number of New 52 Week Highs on US Exchanges
	Bloomberg Total Number of New 52 Week Lows on US Exchanges
	Bloomberg US Composite Stock Exchanges Sentiment High and Low
	Bloomberg Percentage of NYSE Stocks Closing Above 200 Day Moving Average
	Bloomberg Percentage of NASDAQ Stocks Closing Above 200 Day Moving Average
Climate variables	US climate policy
	International summits
	Global warming
	Natural disasters
	Narrative factor US climate policy

Note: The table presents the list of predictors considered in each predictor category including financial, macroeconomic, sentiment, and climate factors.

Table A2: Descriptive statistics of exogenous predictors

Financial variables									
CBOE Oil ETF VIX Index	33.26	10.62	0.79	3.56	261.76	10268.90	19503.37	35374.66	-3.95
CBOE Gold ETF VIX Index	16.65	4.84	1.19	5.19	969.14	9909.92	18645.08	33936.17	-6.23
Chicago Board Options Exchange Volatility Index	16.27	5.33	2.05	8.58	4461.97	8903.23	15780.11	26340.18	-6.84
ICE BofA MOVE Index	69.57	15.54	0.57	2.69	128.85	9970.26	18586.01	32874.54	-5.40
The CBOE Euro Currency Volatility	9.39	2.86	0.83	3.31	265.93	10235.67	19381.24	35293.48	-4.52
J.P. Morgan G7 Volatility Index (VXY)	9.01	1.86	0.28	2.68	39.15	10415.30	20009.93	37269.75	-3.81
Risk aversion index	2.84	0.36	3.59	23.10	42289.95	7903.69	13858.36	22700.85	-8.57
MIAX/T3 Volatility Index	16.50	5.31	2.12	9.06	5080.10	8883.87	15801.60	26393.02	-6.84
GS US Financial Conditions Index	99.53	0.45	0.53	3.59	136.46	10629.61	20492.24	38181.77	-2.83
Macroeconomic variables	Mean	Std. dev	Skewness	Kurtosis	JB	Q(5)	Q(10)	Q(20)	ADF
Morgan Stanley US Shadow Short Rate	-0.73	2.09	-0.12	1.67	169.66	11112.38	22174.01	44120.77	-2.13
Aruoba Diebold Scotti Business Conditions Index »	-0.13	0.31	0.19	2.61	27.81	10659.52	19300.44	28286.41	-3.94
US 10Y Breakeven inflation rate	1.97	0.35	-1.38	9.37	4482.26	6174.10	12105.01	23068.73	-12.68
GS Economic Surprise Index	0.12	0.79	-0.47	3.52	107.92	10369.78	19362.86	32823.19	-3.78
US Economic Policy Uncertainty	99.15	57.74	1.72	7.66	3118.48	3159.46	5254.73	8644.91	-16.94
BDI Baltic Exchange Dry Index	1111.96	425.63	0.65	3.03	158.92	10779.63	20488.67	36172.00	-3.57
Sentiment variables									
Daily News Sentiment Index	0.00	0.14	-0.38	2.90	54.85	10675.80	20312.53	36324.58	-3.10
Citi FX Forecast Uncertainty - USD vs Emerging Markets Currencies	2.77	0.39	0.00	2.29	46.93	10658.33	20336.17	36940.56	-2.75
US Infectious Disease Equity Market Volatility Tracker (EMV) Daily	0.39	0.84	5.93	56.14	275185.38	715.20	997.17	1195.39	-23.75
Nasdaq US Insider Sentiment Index	905.65	285.68	0.15	1.86	129.49	10971.92	21676.27	42333.04	-3.55
Bloomberg Closing ARMs for New York Stock Exchange	1.11	0.54	3.46	26.25	54653.40	6.58	28.44	71.10	-32.65
Bloomberg Closing ARMs for Nasdaq Composite	0.98	0.43	4.31	48.83	201909.65	123.10	160.86	197.31	-26.67
Bloomberg Closing ARMs for American Stock Exchange	1.16	0.71	5.62	87.53	674990.92	11.26	13.61	20.78	-32.87
Bloomberg Total Number of New 52 Week Highs on US Exchanges	400.65	294.17	1.57	6.00	1757.57	4741.94	7139.72	9803.48	-12.78
Bloomberg Total Number of New 52 Week Lows on US Exchanges	356.06	319.14	5.06	39.77	134998.52	3820.88	5132.85	6233.67	-13.86
Bloomberg US Composite Stock Exchanges Sentiment High and Low	0.52	0.22	-0.41	2.24	115.41	6632.41	10701.03	15939.98	-9.91
Bloomberg Percentage of NYSE Stocks Closing Above 200 Day Moving Average	55.20	16.33	-0.71	3.03	187.74	10440.51	19823.37	35714.98	-3.62
Bloomberg Percentage of NASDAQ Stocks Closing Above 200 Day Moving Average	48.90	13.83	-0.53	2.76	110.59	10516.50	20090.29	36548.73	-3.32
Climate risk variables									
US climate policy	0.67	0.87	2.46	12.30	10267.91	525.11	644.28	902.89	-23.82
International summits	0.27	0.45	2.75	13.10	12276.93	167.38	254.30	373.08	-29.09
Global warming	0.34	0.53	3.14	18.32	25448.41	423.92	576.92	664.11	-24.71
Natural disasters	0.25	0.47	3.43	19.73	30371.92	419.35	587.39	770.17	-25.89
Narrative factor US climate policy	-0.06	0.58	-0.65	11.30	6560.39	116.46	152.51	272.06	-27.87

Table A3: The specifications of HAR-DRD models

	Forecasts of realized correlation matrix	Forecasts of realized volatilities	Estimation of coefficients
HARDRD (benchmark)	The forecasts of the realized correlation matrix $\widehat{RC}_{t+1\|t}$ is the moving average of realized correlation matrices over a rolling window of in-sample period (*T1* days).	HAR model	OLS
HARDRD-Finance		HAR-finance model	OLS
LASSO-HARDRD-Finance			Lasso
HARDRD-Macro		HAR-Macro model	OLS
LASSO-HARDRD-Macro			Lasso
HARDRD-Sentiment		HAR-Macro model	OLS
LASSO-HARDRD-Sentiment			Lasso
HARDRD-climate		HAR-climate model	OLS
LASSO-HARDRD-climate			Lasso
HARDRD-volatility		HAR-volatility model	OLS
LASSO-HARDRD-volatility			Lasso
HARDRD-All		HAR-All model	OLS
LASSO-HARDRD-All			Lasso
HARDRD-factor		HAR-factor model	OLS
LASSO-HARDRD-factor			Lasso

Table A4: Robustness check of forecast precision based on the realized kernel estimator.

	RMSE	MCS_TR	MCS_TSQ	Stein loss	MCS_TR	MCS_TSQ
			h=1			
HARDRD	12.4068	0.0011	0.0069	41.8029	0.0012	0.0067
HARDRD-Finance	12.4352	0.0011	0.0069	44.4078	0.0009	0.0032
LASSO-HARDRD-Finance	12.2210	0.0165	0.0165	37.6400	0.0136	0.0644
HARDRD-Macro	12.6562	0.0011	0.0069	44.3477	0.0012	0.0186
LASSO-HARDRD-Macro	12.4637	0.0011	0.0069	39.3920	0.0136	0.0644
HARDRD-Sentiment	12.6641	0.0011	0.0054	44.1564	0.0009	0.0034
LASSO-HARDRD-Sentiment	12.3314	0.0011	0.0069	39.3452	0.0136	0.0612
HARDRD-climate	12.4120	0.0011	0.0069	42.1975	0.0012	0.0125
LASSO-HARDRD-climate	12.2127	1.0000**	1.0000**	37.1295	0.5656**	0.5656**
HARDRD-volatility	12.4517	0.0011	0.0054	45.1216	0.0012	0.0046
LASSO-HARDRD-volatility	12.2616	0.0011	0.0069	36.9962	1.0000**	1.0000**
HARDRD-All	12.6914	0.0011	0.0054	52.3948	0.0012	0.0056
LASSO-HARDRD-All	12.4204	0.0011	0.0069	39.2560	0.0136	0.0644
HARDRD-factor	12.5405	0.0011	0.0054	43.0951	0.0012	0.0056
LASSO-HARDRD-factor	12.3974	0.0011	0.0069	40.6579	0.0012	0.0186
			h=5			
HARDRD	11.8594	0.1462*	0.1344*	34.4639	0.0314	0.0446
HARDRD-Finance	11.8133	0.4493**	0.2772**	36.2049	0.0340	0.0624
LASSO-HARDRD-Finance	11.7831	0.4493**	0.3222**	33.5379	0.0372	0.0812
HARDRD-Macro	11.8671	0.2950**	0.1750*	33.8813	0.0340	0.0695
LASSO-HARDRD-Macro	11.8224	0.4493**	0.2564**	33.6974	0.0340	0.0742
HARDRD-Sentiment	11.8112	0.4493**	0.3222**	32.9295	0.0438	0.1066*
LASSO-HARDRD-Sentiment	11.7451	1.0000**	1.0000**	33.4862	0.0340	0.0742
HARDRD-climate	11.8366	0.4493**	0.2636**	33.4638	0.0340	0.0794
LASSO-HARDRD-climate	11.7537	0.7960**	0.7960**	32.9164	0.0438	0.1066*
HARDRD-volatility	11.8261	0.4493**	0.3222**	32.5686	0.0438	0.1501*
LASSO-HARDRD-volatility	11.7736	0.4493**	0.4711**	30.5981	1.0000**	1.0000**
HARDRD-All	12.0465	0.1462*	0.0896	32.9184	0.2920**	0.3668**
LASSO-HARDRD-All	11.8264	0.4493**	0.2772**	32.3260	0.2851**	0.3165**
HARDRD-factor	11.9032	0.4493**	0.2004*	33.5114	0.0372	0.0913
LASSO-HARDRD-factor	11.8422	0.4493**	0.2564**	30.8060	0.8480**	0.8480**
			h=22			
HARDRD	11.2859	0.0850	0.0400	29.8012	0.0091	0.0071
HARDRD-Finance	11.3245	0.0850	0.0366	29.0227	0.0091	0.0131
LASSO-HARDRD-Finance	11.2492	0.0850	0.0400	29.4471	0.0091	0.0101
HARDRD-Macro	11.4894	0.0679	0.0243	29.1547	0.0091	0.0186
LASSO-HARDRD-Macro	11.4223	0.0850	0.0294	28.5084	0.0865	0.0823
HARDRD-Sentiment	11.4316	0.0679	0.0245	27.8699	0.0865	0.1431*
LASSO-HARDRD-Sentiment	11.3586	0.0850	0.0311	27.5192	0.3765**	0.3018**
HARDRD-climate	11.2870	0.0850	0.0400	28.7211	0.0091	0.0221
LASSO-HARDRD-climate	11.1858	1.0000**	1.0000**	29.2230	0.0197	0.0371
HARDRD-volatility	11.3856	0.0850	0.0400	28.1266	0.3765**	0.3018**
LASSO-HARDRD-volatility	11.3033	0.0850	0.0400	26.7961	1.0000**	1.0000**
HARDRD-All	11.4923	0.0850	0.0311	27.9223	0.3765**	0.2791**
LASSO-HARDRD-All	11.4722	0.0850	0.0311	27.5928	0.3765**	0.3018**
HARDRD-factor	11.3692	0.0850	0.0289	27.2016	0.3765**	0.3438**
LASSO-HARDRD-factor	11.2920	0.0850	0.0400	27.2922	0.3765**	0.3438**

Note: This table presents the pairwise comparisons of the out-of-sample forecasts from the benchmark HARDRD model against the extended model variations based on RMSE and Stein loss functions. The panels report the findings for the one-step, five-step and 22-step forecasts of realized covariance. The lower values of RMSE and Stein loss suggest the higher precision for the realized covariance forecasts. We also report the MCS results to test the significance of differences in forecast precision among different models. The columns denoted by "MCS_TR" and "MCS_TSQ" are the MCS test results given the range statistics,T_R and the semi-quadratic statistics T_{SQ} respectively. * and ** indicate models that are included in the MCS at 10% and 25% confidence levels, respectively.

Table A5: Robustness check of economic analysis based on the realized kernel estimator.

				c =0%	c =1%	c =2%	c =0%	c =1%	c =2%	γ=1			γ=10		
	GMVP	Turnover ratio	CO	Sharpe1	Sharpe2	Sharpe3	return1	return2	return3	EV1_1	EV1_2	EV1_3	EV2_1	EV2_2	EV2_3
								h=1							
HARDRD	0.1651	0.7653	0.5939	0.0562	0.0444	0.0325	9.1581	7.2296	5.3011	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARDRD-Finance	0.1665	0.7711	0.5949	0.0564	0.0445	0.0325	9.1919	7.2488	5.3057	0.0469	0.0258	0.0048	0.0283	0.0078	-0.0128
LASSO-HARDRD-Finance	0.1580	0.6431	0.5561	0.0532	0.0432	0.0332	8.6395	7.0189	5.3983	-0.7460	-0.2999	0.1462	-0.6976	-0.2500	0.1976
HARDRD-Macro	0.1833	0.4456	0.5636	0.0491	0.0422	0.0353	7.9850	6.8619	5.7389	-1.6921	-0.5249	0.6423	-1.6243	-0.4528	0.7193
LASSO-HARDRD-Macro	0.1700	0.5779	0.5392	0.0527	0.0438	0.0348	8.5722	7.1159	5.6597	-0.8460	-0.1615	0.5230	-0.8203	-0.1321	0.5564
HARDRD-Sentiment	0.1765	0.5042	0.5857	0.0548	0.0469	0.0391	8.8432	7.5727	6.3022	-0.4380	0.5157	1.4694	-0.2666	0.6928	1.6527
LASSO-HARDRD-Sentiment	0.1630	0.5852	0.5534	0.0548	0.0457	0.0365	8.8390	7.3643	5.8896	-0.4468	0.2110	0.8687	-0.3009	0.3607	1.0226
HARDRD-climate	0.1658	0.7757	0.5967	0.0527	0.0407	0.0287	8.5678	6.6131	4.6584	-0.8513	-0.8892	-0.9271	-0.8163	-0.8535	-0.8906
LASSO-HARDRD-climate	0.1579	0.6338	0.5553	0.0506	0.0408	0.0310	8.2278	6.6307	5.0336	-1.3441	-0.8639	-0.3838	-1.3118	-0.8310	-0.3500
HARDRD-volatility	0.1725	0.3702	0.5590	0.0518	0.0460	0.0403	8.4176	7.4846	6.5517	-1.0684	0.3739	1.8163	-1.0289	0.4163	1.8623
LASSO-HARDRD-volatility	0.1659	0.4046	0.5174	0.0450	0.0387	0.0324	7.2952	6.2758	5.2563	-2.6912	-1.3744	-0.0575	-2.6245	-1.3065	0.0123
HARDRD-All	0.1721	0.4935	0.6011	0.0427	0.0353	0.0279	7.1833	5.9398	4.6963	-2.9064	-1.9136	-0.9207	-3.3403	-2.3419	-1.3430
LASSO-HARDRD-All	0.1709	0.4516	0.5196	0.0484	0.0414	0.0343	7.7888	6.6508	5.5127	-1.9617	-0.8167	0.3284	-1.7570	-0.6112	0.5352
HARDRD-factor	0.1771	0.3365	0.5536	0.0543	0.0491	0.0438	8.8036	7.9557	7.1077	-0.5013	1.0643	2.6300	-0.3857	1.1840	2.7546
LASSO-HARDRD-factor	0.1689	0.5422	0.5325	0.0525	0.0440	0.0355	8.4689	7.1025	5.7361	-0.9831	-0.1688	0.6455	-0.8395	-0.0244	0.7910
								h=5							
HARDRD	0.2000	0.2256	0.6076	0.1264	0.1184	0.1105	9.1078	8.5392	7.9706	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARDRD-Finance	0.1960	0.2180	0.6110	0.1297	0.1221	0.1145	9.3668	8.8174	8.2680	0.3751	0.4030	0.4309	0.3751	0.4038	0.4324
LASSO-HARDRD-Finance	0.1965	0.2023	0.6004	0.1229	0.1158	0.1087	8.8082	8.2984	7.7886	-0.4323	-0.3471	-0.2619	-0.4160	-0.3301	-0.2441
HARDRD-Macro	0.2023	0.1755	0.6014	0.1115	0.1053	0.0991	7.9653	7.5230	7.0808	-1.6517	-1.4687	-1.2857	-1.6225	-1.4388	-1.2550
LASSO-HARDRD-Macro	0.1967	0.1867	0.5908	0.1211	0.1146	0.1080	8.7312	8.2607	7.7901	-0.5447	-0.4026	-0.2605	-0.5379	-0.3947	-0.2515
HARDRD-Sentiment	0.2025	0.2000	0.6080	0.1259	0.1190	0.1121	9.2024	8.6983	8.1942	0.1340	0.2275	0.3210	0.1066	0.2008	0.2950
LASSO-HARDRD-Sentiment	0.1979	0.2063	0.6008	0.1239	0.1167	0.1095	8.9394	8.4195	7.8997	-0.2443	-0.1735	-0.1027	-0.2468	-0.1742	-0.1016
HARDRD-climate	0.2017	0.2389	0.6101	0.1289	0.1205	0.1122	9.3034	8.7013	8.0993	0.2826	0.2342	0.1857	0.2765	0.2281	0.1796
LASSO-HARDRD-climate	0.1969	0.1626	0.5902	0.1188	0.1131	0.1073	8.5190	8.1094	7.6997	-0.8507	-0.6204	-0.3901	-0.8306	-0.5991	-0.3676
HARDRD-volatility	0.2037	0.2116	0.6025	0.1197	0.1123	0.1049	8.6222	8.0890	7.5559	-0.7028	-0.6514	-0.6001	-0.6961	-0.6449	-0.5936
LASSO-HARDRD-volatility	0.1961	0.1708	0.5893	0.1144	0.1084	0.1025	8.2451	7.8147	7.3843	-1.2489	-1.0486	-0.8483	-1.2423	-1.0408	-0.8393
HARDRD-All	0.2067	0.3023	0.6163	0.1245	0.1140	0.1036	9.0477	8.2860	7.5242	-0.0904	-0.3701	-0.6497	-0.1218	-0.4005	-0.6792

LASSO-HARDRD-All	0.1974	0.2527	0.6055	0.1300	0.1211	0.1122	9.3488	8.7121	8.0754	0.3511	0.2525	0.1539	0.3707	0.2728	0.1748
HARDRD-factor	0.2026	0.2287	0.6020	0.1153	0.1073	0.0992	8.3036	7.7272	7.1509	-1.1626	-1.1739	-1.1852	-1.1411	-1.1529	-1.1647
LASSO-HARDRD-factor	0.2003	0.1783	0.5886	0.1145	0.1083	0.1020	8.2219	7.7727	7.3234	-1.2809	-1.1078	-0.9348	-1.2590	-1.0848	-0.9105
h=22															
HARDRD	0.2238	0.1521	0.6175	0.2709	0.2583	0.2457	8.2276	7.8442	7.4608	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARDRD-Finance	0.2179	0.1593	0.6389	0.2410	0.2281	0.2152	7.4914	7.0900	6.6887	-1.0676	-1.0936	-1.1197	-1.0801	-1.1061	-1.1321
LASSO-HARDRD-Finance	0.2200	0.1636	0.6136	0.2510	0.2376	0.2242	7.6997	7.2874	6.8751	-0.7651	-0.8070	-0.8490	-0.7703	-0.8128	-0.8553
HARDRD-Macro	0.2286	0.1339	0.6415	0.2478	0.2368	0.2258	7.6152	7.2778	6.9404	-0.8874	-0.8208	-0.7542	-0.8911	-0.8246	-0.7582
LASSO-HARDRD-Macro	0.2295	0.1494	0.6215	0.2445	0.2321	0.2198	7.4938	7.1175	6.7411	-1.0629	-1.0528	-1.0427	-1.0648	-1.0551	-1.0453
HARDRD-Sentiment	0.2288	0.1536	0.6261	0.2517	0.2392	0.2267	7.7922	7.4051	7.0179	-0.6318	-0.6373	-0.6428	-0.6440	-0.6498	-0.6556
LASSO-HARDRD-Sentiment	0.2274	0.1607	0.6152	0.2804	0.2670	0.2536	8.4394	8.0344	7.6295	0.3079	0.2766	0.2454	0.3174	0.2863	0.2553
HARDRD-climate	0.2274	0.1580	0.6141	0.2836	0.2704	0.2573	8.5819	8.1838	7.7856	0.5136	0.4923	0.4709	0.5173	0.4958	0.4742
LASSO-HARDRD-climate	0.2187	0.1508	0.6106	0.2559	0.2435	0.2312	7.8826	7.5025	7.1224	-0.5003	-0.4956	-0.4908	-0.5067	-0.5024	-0.4980
HARDRD-volatility	0.2328	0.1524	0.6279	0.2933	0.2804	0.2675	8.7101	8.3259	7.9418	0.6992	0.6981	0.6969	0.7029	0.7009	0.6988
LASSO-HARDRD-volatility	0.2259	0.1652	0.6015	0.2643	0.2504	0.2364	7.9476	7.5314	7.1151	-0.4053	-0.4530	-0.5007	-0.4041	-0.4524	-0.5008
HARDRD-All	0.2227	0.1916	0.6552	0.2313	0.2162	0.2010	7.3507	6.8679	6.3851	-1.2704	-1.4145	-1.5587	-1.2735	-1.4192	-1.5648
LASSO-HARDRD-All	0.2240	0.1806	0.6447	0.2313	0.2169	0.2025	7.3076	6.8525	6.3974	-1.3335	-1.4376	-1.5416	-1.3436	-1.4491	-1.5547
HARDRD-factor	0.2177	0.1345	0.6466	0.2440	0.2332	0.2223	7.6260	7.2872	6.9483	-0.8729	-0.8085	-0.7440	-0.8888	-0.8246	-0.7605
LASSO-HARDRD-factor	0.2219	0.1575	0.6102	0.2496	0.2367	0.2237	7.6680	7.2712	6.8744	-0.8102	-0.8296	-0.8490	-0.8078	-0.8273	-0.8468

Note: The table reports the economic evaluation results for the constructed industry portfolios obtained from competing forecasting models for h-step ahead forecasts. We report the volatility forecast for the Global Minimum Variance Portfolio (GMVP), the average turnover ratio, the average concentration ration (CO), annualized average out-of-sample return and the Sharpe Ratios (SR) based on three alternative transaction costs, c= 0, 1%, 2%. We also report the Economic Value (EV) of the constructed portfolios over the benchmark HARDRD model obtained using two risk aversion rates, γ=1 and γ=10. Lower values for the GMVP, the average turnover ratio, the average concentration ration (CO) indicate improved portfolio performance.

Table A6: Beta-neutral portfolios based on the realized kernel estimator.

	h=1		h=5		h=22	
	beta_neutral	t_statistics	beta_neutral	t_statistics	beta_neutral	t_statistics
HARDRD	0.1031***	6.3540	0.0678***	3.6501	0.0991***	5.7791
HARDRD-Finance	0.1051***	6.6600	0.0780***	4.2000	0.0877***	4.9534
LASSO-HARDRD-Finance	0.0759***	4.6656	0.0708***	3.6275	0.0744***	4.1792
HARDRD-Macro	0.0821***	5.2432	0.0576***	3.1470	0.0888***	5.1114
LASSO-HARDRD-Macro	0.0634***	3.7963	0.0523***	2.8393	0.0819***	4.6242
HARDRD-Sentiment	0.0600***	3.6445	0.0773***	3.9505	0.0902***	4.9489
LASSO-HARDRD-Sentiment	0.0701***	4.1527	0.0656***	3.4001	0.0818***	4.4106
HARDRD-climate	0.1146***	7.3601	0.0664***	3.5675	0.0947***	5.3744
LASSO-HARDRD-climate	0.0743***	4.4595	0.0424**	2.2267	0.0720***	3.9489
HARDRD-volatility	0.0783***	4.6640	0.0666***	3.6364	0.0927***	5.0587
LASSO-HARDRD-volatility	0.0342**	2.0453	0.0463***	2.4203	0.0738***	4.0849
HARDRD-All	0.1322***	7.6499	0.0961***	4.9621	0.0974***	5.5038
LASSO-HARDRD-All	0.0243	1.4480	0.0583***	3.0020	0.0904***	5.0728
HARDRD-factor	0.0474***	2.6774	0.0626***	3.3033	0.1072***	6.2571
LASSO-HARDRD-factor	0.0458***	2.6176	0.0575***	3.0354	0.0767***	4.2114

Note: This table presents the actual betas of beta-neutral portfolios constructed by competing forecasting models for h-step ahead forecasts. For each forecast approach, we sort the 10 industry indices into high and low beta groups based on their respective current beta forecasts ($\hat{\beta}$) according to Equation 31. We then construct the market-neutral portfolios ex ante by solving the equation $\upsilon_{j,t}\hat{\beta}_{long\ j,t} - \hat{\beta}_{short,j,t} = 0$ and applying the resulting weight $\upsilon_{j,t}$ to the long leg of the portfolios. We also compute the t-statistics according to the robust Newey and West (1987) standard errors with 6 lags, in order to test whether the actual beta of the portfolio is significantly different from 0. An estimated portfolio beta value close to 0 and insignificant t-statistic imply that the portfolios is market-neutral.

ONLINE APPENDIX

Table OA1: Robustness check of forecast precision with alternative forecast horizon

	RMSE	MCS_TR	MCS_TSQ	Stein loss	MCS_TR	MCS_TSQ
			h=1			
HARDRD	8.1364	0.0002	0.0031	21.5169	0.0001	0.0030
HARDRD-Finance	8.1645	0.0002	0.0031	22.6430	0.0001	0.0019
LASSO-HARDRD-Finance	7.8357	0.0027	0.0044	19.3341	0.0096	0.0366
HARDRD-Macro	8.7909	0.0002	0.0031	23.7140	0.0096	0.0102
LASSO-HARDRD-Macro	8.1659	0.0027	0.0044	20.5850	0.0096	0.0140
HARDRD-Sentiment	8.8156	0.0002	0.0031	22.2826	0.0096	0.0109
LASSO-HARDRD-Sentiment	7.9136	0.0027	0.0044	19.8477	0.0096	0.0140
HARDRD-climate	8.1582	0.0002	0.0031	21.7414	0.0001	0.0015
LASSO-HARDRD-climate	7.8262	1.0000**	1.0000**	19.2238	0.8244**	0.8244**
HARDRD-volatility	8.2347	0.0002	0.0031	22.7057	0.0001	0.0030
LASSO-HARDRD-volatility	7.8832	0.0027	0.0044	19.1886	1.0000**	1.0000**
HARDRD-All	9.5148	0.0002	0.0031	24.0874	0.0001	0.0012
LASSO-HARDRD-All	8.1587	0.0027	0.0044	19.9848	0.0096	0.0366
HARDRD-factor	8.2432	0.0002	0.0031	22.5053	0.0001	0.0021
LASSO-HARDRD-factor	7.9479	0.0027	0.0044	20.1076	0.0096	0.0140
			h=5			
HARDRD	7.8162	0.0019	0.0120	18.8710	0.0007	0.0118
HARDRD-Finance	7.8020	0.0124	0.0171	19.2739	0.0192	0.0192
LASSO-HARDRD-Finance	7.6739	0.2070*	0.2070*	18.0227	0.1529*	0.1112*
HARDRD-Macro	7.8563	0.0124	0.0219	18.7144	0.0192	0.0282
LASSO-HARDRD-Macro	7.7409	0.1878*	0.1458*	18.2942	0.0192	0.0344
HARDRD-Sentiment	7.8871	0.0019	0.0118	18.6869	0.0192	0.0192
LASSO-HARDRD-Sentiment	7.7489	0.1878*	0.1458*	17.9186	0.1529*	0.1115*
HARDRD-climate	7.7979	0.0124	0.0404	18.3301	0.0192	0.0221
LASSO-HARDRD-climate	7.6573	1.0000**	1.0000**	17.6707	0.1529*	0.1507*
HARDRD-volatility	7.8119	0.0124	0.0221	18.3065	0.0370	0.0660
LASSO-HARDRD-volatility	7.7168	0.1878*	0.1458*	17.2010	1.0000**	1.0000**
HARDRD-All	7.9424	0.0019	0.0088	22.3178	0.0007	0.0106
LASSO-HARDRD-All	7.7637	0.1878*	0.1458*	19.1260	0.0185	0.0151
HARDRD-factor	7.9011	0.0019	0.0064	18.7120	0.0192	0.0221
LASSO-HARDRD-factor	7.8017	0.0124	0.0205	17.3525	0.6846**	0.6846**
			h=22			
HARDRD	7.5281	0.0022	0.0050	16.4878	0.0002	0.0320
HARDRD-Finance	7.6106	0.0012	0.0044	16.1994	0.0002	0.0648
LASSO-HARDRD-Finance	7.4984	0.0059	0.0055	16.0572	0.0002	0.0648
HARDRD-Macro	7.6709	0.0008	0.0044	16.4079	0.0002	0.0648
LASSO-HARDRD-Macro	7.6459	0.0008	0.0044	16.1950	0.0002	0.0648
HARDRD-Sentiment	7.6805	0.0008	0.0015	16.6280	0.0002	0.0104
LASSO-HARDRD-Sentiment	7.6662	0.0008	0.0032	16.3348	0.0002	0.0407
HARDRD-climate	7.5344	0.0059	0.0055	16.0820	0.0159	0.1289*
LASSO-HARDRD-climate	7.4362	1.0000**	1.0000**	15.7348	1.0000**	1.0000**
HARDRD-volatility	7.6785	0.0008	0.0018	17.6051	0.0002	0.0164
LASSO-HARDRD-volatility	7.6141	0.0008	0.0044	15.9577	0.6193**	0.6122**
HARDRD-All	7.6719	0.0022	0.0044	16.7748	0.0002	0.0241
LASSO-HARDRD-All	7.6699	0.0008	0.0044	16.4757	0.0002	0.0410
HARDRD-factor	7.5850	0.0022	0.0050	16.2069	0.0002	0.0648
LASSO-HARDRD-factor	7.5245	0.0059	0.0055	15.8545	0.6193**	0.6122**

Note: This table presents the pairwise comparisons of the out-of-sample forecasts from the benchmark HARDRD model against the extended model variations based on RMSE and Stein loss functions. The panels report the findings for the one-step, five-step and 22-step forecasts of realized covariance. The lower values of RMSE and Stein loss suggest the higher precision for the realized covariance forecasts. We also report the MCS results to test the significance of differences in forecast precision among different models. The columns denoted by "MCS_TR" and "MCS_TSQ" are the MCS test results given the range statistics, T_R and the semi-quadratic statistics T_{SQ} respectively. We denote the models that are included in the MCS at 10% and 25% confidence levels with * and ** respectively.

Table OA2: Robustness check of portfolio analysis with alternative forecast horizon.

				c =0	c =1%	c =2%	c =0	c =1%	c =2%	γ=1			γ=10		
	GMVP	Turnover ratio	CO	Sharpe1	Sharpe2	Sharpe3	return1	return2	return3	EV1_1	EV1_2	EV1_3	EV2_1	EV2_2	EV2_3
								h=1							
HARDRD	0.1776	0.6735	0.5890	0.0434	0.0335	0.0236	7.4408	5.7435	4.0462	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARDRD-Finance	0.1823	0.6781	0.5924	0.0436	0.0336	0.0237	7.5006	5.7918	4.0830	0.0813	0.0646	0.0479	0.0313	0.0150	-0.0014
LASSO-HARDRD-Finance	0.1719	0.5839	0.5582	0.0434	0.0348	0.0262	7.4401	5.9687	4.4974	0.0015	0.3290	0.6566	0.0251	0.3551	0.6851
HARDRD-Macro	0.2042	0.4343	0.5921	0.0461	0.0397	0.0334	7.9051	6.8105	5.7159	0.6758	1.5494	2.4229	0.7071	1.5855	2.4643
LASSO-HARDRD-Macro	0.1830	0.5332	0.5476	0.0472	0.0394	0.0315	8.0954	6.7517	5.4080	0.9525	1.4649	1.9774	0.9936	1.5084	2.0233
HARDRD-Sentiment	0.1919	0.4970	0.5878	0.0416	0.0343	0.0270	7.1351	5.8826	4.6301	-0.4432	0.2017	0.8466	-0.4465	0.2038	0.8544
LASSO-HARDRD-Sentiment	0.1743	0.5594	0.5571	0.0418	0.0336	0.0254	7.1637	5.7541	4.3444	-0.3982	0.0188	0.4358	-0.3689	0.0511	0.4712
HARDRD-climate	0.1802	0.6947	0.5897	0.0420	0.0318	0.0216	7.2276	5.4769	3.7263	-0.3146	-0.3919	-0.4692	-0.3677	-0.4453	-0.5229
LASSO-HARDRD-climate	0.1716	0.5783	0.5559	0.0427	0.0341	0.0256	7.2935	5.8362	4.3790	-0.2064	0.1416	0.4895	-0.1416	0.2089	0.5596
HARDRD-volatility	0.1934	0.3909	0.5654	0.0364	0.0307	0.0250	6.2820	5.2969	4.3119	-1.6906	-0.6588	0.3731	-1.8033	-0.7695	0.2647
LASSO-HARDRD-volatility	0.1794	0.4104	0.5260	0.0431	0.0370	0.0310	7.3646	6.3304	5.2962	-0.1040	0.8569	1.8178	-0.0443	0.9201	1.8849
HARDRD-All	0.1910	0.5870	0.6265	0.0434	0.0350	0.0266	7.6325	6.1533	4.6740	0.2286	0.5446	0.8606	-0.2254	0.0925	0.4104
LASSO-HARDRD-All	0.1846	0.4656	0.5328	0.0415	0.0347	0.0279	7.1397	5.9664	4.7931	-0.4381	0.3211	1.0803	-0.4558	0.3050	1.0661
HARDRD-factor	0.1843	0.3757	0.5656	0.0399	0.0345	0.0290	6.8947	5.9480	5.0013	-0.8008	0.2866	1.3740	-0.8919	0.1974	1.2872
LASSO-HARDRD-factor	0.1765	0.5291	0.5432	0.0424	0.0346	0.0269	7.3114	5.9779	4.6444	-0.1972	0.3298	0.8568	-0.2869	0.2400	0.7671
								h=5							
HARDRD	0.2022	0.1892	0.5917	0.0831	0.0767	0.0703	6.1809	5.7042	5.2274	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARDRD-Finance	0.1998	0.1879	0.5940	0.0879	0.0815	0.0751	6.5310	6.0575	5.5841	0.5067	0.5115	0.5163	0.5040	0.5092	0.5144
LASSO-HARDRD-Finance	0.1980	0.1776	0.5814	0.0910	0.0850	0.0789	6.7396	6.2921	5.8446	0.8112	0.8536	0.8961	0.8300	0.8726	0.9153
HARDRD-Macro	0.2035	0.1784	0.5875	0.0777	0.0717	0.0656	5.7854	5.3359	4.8864	-0.5744	-0.5349	-0.4953	-0.5877	-0.5479	-0.5081
LASSO-HARDRD-Macro	0.1966	0.2295	0.5817	0.0736	0.0658	0.0581	5.4884	4.9100	4.3317	-1.0059	-1.1532	-1.3005	-1.0318	-1.1808	-1.3298
HARDRD-Sentiment	0.2055	0.1971	0.5992	0.0840	0.0774	0.0707	6.2399	5.7432	5.2464	0.0851	0.0562	0.0273	0.0821	0.0537	0.0253
LASSO-HARDRD-Sentiment	0.2004	0.2212	0.5884	0.0807	0.0733	0.0659	6.0554	5.4980	4.9405	-0.1866	-0.3035	-0.4205	-0.2305	-0.3484	-0.4662
HARDRD-climate	0.2046	0.2082	0.5940	0.0868	0.0798	0.0728	6.4917	5.9671	5.4425	0.4477	0.3784	0.3091	0.4256	0.3560	0.2865
LASSO-HARDRD-climate	0.1984	0.1510	0.5772	0.0896	0.0845	0.0794	6.6327	6.2522	5.8717	0.6576	0.7970	0.9364	0.6877	0.8271	0.9666
HARDRD-volatility	0.1978	0.1906	0.5955	0.0852	0.0788	0.0723	6.3169	5.8366	5.3564	0.1992	0.1942	0.1892	0.2209	0.2160	0.2110
LASSO-HARDRD-volatility	0.2018	0.1702	0.5669	0.0858	0.0800	0.0742	6.3406	5.9118	5.4830	0.2349	0.3044	0.3738	0.2687	0.3378	0.4068
HARDRD-All	0.1976	0.3810	0.6275	0.0969	0.0842	0.0715	7.3106	6.3505	5.3903	1.6307	0.9303	0.2299	1.5799	0.8769	0.1740
LASSO-HARDRD-All	0.1973	0.3469	0.6042	0.0823	0.0707	0.0590	6.1630	5.2887	4.4145	-0.0303	-0.6062	-1.1821	-0.0702	-0.6479	-1.2255
HARDRD-factor	0.2076	0.2625	0.5893	0.0980	0.0891	0.0803	7.3045	6.6431	5.9817	1.6297	1.3619	1.0942	1.6505	1.3802	1.1099
LASSO-HARDRD-factor	0.2036	0.1707	0.5728	0.0939	0.0881	0.0823	6.9458	6.5158	6.0857	1.1115	1.1791	1.2467	1.1447	1.2120	1.2794
								h=22							
HARDRD	0.2239	0.1205	0.5996	0.1939	0.1843	0.1746	6.1180	5.8145	5.5109	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARDRD-Finance	0.2244	0.1334	0.6098	0.1976	0.1870	0.1763	6.2374	5.9012	5.5651	0.1737	0.1264	0.0792	0.1816	0.1339	0.0861
LASSO-HARDRD-Finance	0.2215	0.1406	0.5989	0.1892	0.1780	0.1668	5.9822	5.6279	5.2736	-0.1967	-0.2703	-0.3439	-0.1966	-0.2712	-0.3458
HARDRD-Macro	0.2275	0.1296	0.6081	0.1821	0.1719	0.1616	5.7946	5.4681	5.1416	-0.4704	-0.5039	-0.5373	-0.4885	-0.5236	-0.5587
LASSO-HARDRD-Macro	0.2272	0.1346	0.6088	0.1876	0.1770	0.1665	6.0315	5.6924	5.3533	-0.1268	-0.1784	-0.2301	-0.1404	-0.1937	-0.2470
HARDRD-Sentiment	0.2271	0.1385	0.6212	0.2021	0.1911	0.1801	6.3495	6.0006	5.6516	0.3348	0.2689	0.2030	0.3297	0.2628	0.1958

LASSO-HARDRD-Sentiment	0.2288	0.1566	0.6020	0.2026	0.1902	0.1778	6.3679	5.9733	5.5788	0.3621	0.2301	0.0981	0.3639	0.2300	0.0961
HARDRD-climate	0.2255	0.1397	0.6017	0.1992	0.1881	0.1770	6.3180	5.9660	5.6139	0.2892	0.2188	0.1485	0.2850	0.2136	0.1422
LASSO-HARDRD-climate	0.2231	0.1329	0.5899	0.1836	0.1731	0.1625	5.7954	5.4605	5.1256	-0.4677	-0.5133	-0.5589	-0.4719	-0.5188	-0.5656
HARDRD-volatility	0.2244	0.1308	0.6307	0.2062	0.1956	0.1850	6.4598	6.1302	5.8005	0.4958	0.4577	0.4196	0.5018	0.4616	0.4214
LASSO-HARDRD-volatility	0.2244	0.1252	0.6229	0.2063	0.1961	0.1860	6.4382	6.1226	5.8070	0.4648	0.4472	0.4296	0.4746	0.4556	0.4366
HARDRD-All	0.2254	0.1726	0.6135	0.2051	0.1914	0.1776	6.4713	6.0364	5.6014	0.5120	0.3213	0.1306	0.5145	0.3204	0.1262
LASSO-HARDRD-All	0.2263	0.1559	0.6129	0.2132	0.2008	0.1883	6.7226	6.3298	5.9370	0.8761	0.7466	0.6170	0.8806	0.7480	0.6154
HARDRD-factor	0.2196	0.1247	0.6131	0.1969	0.1869	0.1768	6.1713	5.8571	5.5428	0.0783	0.0626	0.0469	0.0879	0.0709	0.0538
LASSO-HARDRD-factor	0.2234	0.1254	0.5866	0.1967	0.1867	0.1766	6.1503	5.8343	5.5184	0.0480	0.0300	0.0119	0.0602	0.0410	0.0217

Note: The table reports the economic evaluation results for h-step ahead forecasts obtained from competing forecasting models. We report the GMVP, the average turnover ratio, the average concentration ration (CO), and annualized average out-of-sample return given three transaction cost rate c= 0, 1%, 2%. We also report the Sharpe Ratios (SR) which are computed based on three different transaction costs, i.e. (c= 0, 1%, 2%) as well as the Economic Value (EV) of the portfolios are over the benchmark HARDRD model is obtained using two risk aversion rates, γ=1 and γ=10.

Table OA3: Robustness check of forecast precision with alternative forecast window (126 days).

	RMSE	MCS_TR	MCS_TSQ	Stein loss	MCS_TR	MCS_TSQ
			h=1			
HARDRD	12.4268	0.0015	0.0073	29.2548	0.0037	0.0298
HARDRD-Finance	12.4491	0.0015	0.0073	30.5733	0.0011	0.0180
LASSO-HARDRD-Finance	12.2497	0.0262	0.0262	27.6814	0.1395*	0.2311*
HARDRD-Macro	12.5868	0.0015	0.0073	31.6049	0.0037	0.0298
LASSO-HARDRD-Macro	12.4486	0.0015	0.0073	29.4256	0.0037	0.0298
HARDRD-Sentiment	12.6146	0.0015	0.0073	30.4133	0.0011	0.0195
LASSO-HARDRD-Sentiment	12.3325	0.0015	0.0073	28.3558	0.1395*	0.0938
HARDRD-climate	12.4340	0.0015	0.0073	29.2264	0.0270	0.0517
LASSO-HARDRD-climate	12.2447	1.0000**	1.0000**	27.6220	1.0000**	1.0000**
HARDRD-volatility	12.4704	0.0015	0.0064	31.1014	0.0011	0.0180
LASSO-HARDRD-volatility	12.2805	0.0015	0.0073	27.6481	0.7211**	0.7211**
HARDRD-All	12.5915	0.0015	0.0064	34.9847	0.0014	0.0214
LASSO-HARDRD-All	12.4208	0.0015	0.0073	29.2643	0.1395*	0.1330*
HARDRD-factor	12.5095	0.0015	0.0073	30.1121	0.0014	0.0214
LASSO-HARDRD-factor	12.3568	0.0015	0.0073	28.8042	0.0037	0.0298
			h=5			
HARDRD	11.9001	0.1803*	0.1075*	25.1024	0.0049	0.0620
HARDRD-Finance	11.8560	0.2673**	0.1759*	26.6366	0.0558	0.1165*
LASSO-HARDRD-Finance	11.8297	0.2673**	0.1779*	24.4587	0.2404*	0.2044*
HARDRD-Macro	11.9114	0.1803*	0.0992	24.7751	0.0892	0.1720*
LASSO-HARDRD-Macro	11.8671	0.2673**	0.1579*	24.8328	0.0558	0.1374*
HARDRD-Sentiment	11.8398	0.2673**	0.2127*	24.3621	0.2404*	0.2120*
LASSO-HARDRD-Sentiment	11.7716	1.0000**	1.0000**	24.4342	0.2404*	0.2044*
HARDRD-climate	11.8918	0.2673**	0.1626*	24.1701	0.2404*	0.2120*
LASSO-HARDRD-climate	11.7976	0.5626**	0.5626**	24.1768	0.2404*	0.2120*
HARDRD-volatility	11.8711	0.2673**	0.1768*	24.9791	0.2404*	0.2044*
LASSO-HARDRD-volatility	11.8209	0.2673**	0.2783**	23.7141	0.5061**	0.5061**
HARDRD-All	12.0793	0.1803*	0.0676	25.5432	0.0558	0.1003*
LASSO-HARDRD-All	11.8750	0.2673**	0.1759*	24.2480	0.2404*	0.2120*
HARDRD-factor	11.9526	0.2673**	0.1312*	25.0694	0.0892	0.1710*
LASSO-HARDRD-factor	11.8916	0.2673**	0.1579*	23.2232	1.0000**	1.0000**
			h=22			
HARDRD	11.3868	0.0709	0.0390	22.0390	0.0026	0.0537
HARDRD-Finance	11.4108	0.0709	0.0315	21.6993	0.0026	0.0688
LASSO-HARDRD-Finance	11.3312	0.0709	0.0390	21.8887	0.0026	0.0876
HARDRD-Macro	11.5599	0.0140	0.0151	21.5110	0.0909	0.2315*
LASSO-HARDRD-Macro	11.4915	0.0140	0.0196	21.3392	0.0909	0.2315*
HARDRD-Sentiment	11.5269	0.0140	0.0151	21.4527	0.0026	0.1271*
LASSO-HARDRD-Sentiment	11.4587	0.0140	0.0207	21.0441	0.0909	0.2315*
HARDRD-climate	11.3849	0.0709	0.0390	21.1111	0.1466*	0.2315*
LASSO-HARDRD-climate	11.2859	1.0000**	1.0000**	21.4680	0.0909	0.2315*
HARDRD-volatility	11.4732	0.0709	0.0340	21.4314	0.0909	0.2315*
LASSO-HARDRD-volatility	11.3919	0.0709	0.0390	20.6414	1.0000**	1.0000**
HARDRD-All	11.5485	0.0709	0.0271	21.4188	0.0909	0.2315*
LASSO-HARDRD-All	11.5419	0.0140	0.0207	21.2843	0.0909	0.2315*
HARDRD-factor	11.4517	0.0140	0.0171	20.9690	0.2042*	0.2315*
LASSO-HARDRD-factor	11.3791	0.0709	0.0390	20.8601	0.2460*	0.2460*

Note: This table presents the pairwise comparisons of the out-of-sample forecasts from the benchmark HARDRD model against the extended model variations based on RMSE and Stein loss functions. The panels report the findings for the one-step, five-step and 22-step forecasts of realized covariance. The lower values of RMSE and Stein loss suggest the higher precision for the realized covariance forecasts. We also report the MCS results to test the significance of differences in forecast precision among different models. The columns denoted by "MCS_TR" and "MCS_TSQ" are the MCS test results given the range statistics,T_R and the semi-quadratic statistics T_{SQ} respectively. We denote the models that are included in the MCS at 10% and 25% confidence levels with * and ** respectively.

Table OA4: Robustness check of forecast precision with alternative forecast window (252 days).

	RMSE	MCS_TR	MCS_TSQ	Stein loss	MCS_TR	MCS_TSQ
			h=1			
HARDRD	12.4504	0.0023	0.0083	26.8558	0.0312	0.0677
HARDRD-Finance	12.4726	0.0023	0.0083	28.0449	0.0010	0.0176
LASSO-HARDRD-Finance	12.2692	0.0297	0.0297	25.3678	0.2755**	0.4173**
HARDRD-Macro	12.6211	0.0023	0.0083	28.4835	0.0312	0.0644
LASSO-HARDRD-Macro	12.4747	0.0023	0.0083	26.6318	0.0312	0.0791
HARDRD-Sentiment	12.6472	0.0023	0.0083	27.9409	0.0010	0.0198
LASSO-HARDRD-Sentiment	12.3564	0.0023	0.0083	25.9576	0.0312	0.0791
HARDRD-climate	12.4592	0.0023	0.0083	26.9291	0.0312	0.0791
LASSO-HARDRD-climate	12.2642	1.0000**	1.0000**	25.3095	1.0000**	1.0000**
HARDRD-volatility	12.4924	0.0023	0.0063	28.5546	0.0010	0.0153
LASSO-HARDRD-volatility	12.2995	0.0023	0.0083	25.3288	0.8688**	0.8688**
HARDRD-All	12.6132	0.0023	0.0063	31.7223	0.0312	0.0513
LASSO-HARDRD-All	12.4459	0.0023	0.0083	26.5675	0.2755**	0.2603**
HARDRD-factor	12.5368	0.0023	0.0083	27.8278	0.0021	0.0258
LASSO-HARDRD-factor	12.3781	0.0023	0.0083	26.2063	0.0312	0.0791
			h=5			
HARDRD	11.9144	0.1562*	0.0958	24.3784	0.0204	0.0440
HARDRD-Finance	11.8717	0.2455*	0.1513*	25.7233	0.0574	0.0836
LASSO-HARDRD-Finance	11.8456	0.2455*	0.1560*	23.8084	0.0574	0.0886
HARDRD-Macro	11.9231	0.1562*	0.0913	23.6700	0.0574	0.0886
LASSO-HARDRD-Macro	11.8798	0.2455*	0.1466*	23.7415	0.0574	0.0886
HARDRD-Sentiment	11.8493	0.2455*	0.1998*	23.6916	0.0907	0.0886
LASSO-HARDRD-Sentiment	11.7821	1.0000**	1.0000**	23.8163	0.0574	0.0886
HARDRD-climate	11.9047	0.2455*	0.1466*	23.4681	0.0907	0.0886
LASSO-HARDRD-climate	11.8128	0.4986**	0.4986**	23.3730	0.0907	0.0886
HARDRD-volatility	11.8884	0.2455*	0.1513*	24.3951	0.0574	0.0886
LASSO-HARDRD-volatility	11.8360	0.2455*	0.2312*	22.7295	0.4784**	0.4784**
HARDRD-All	12.0984	0.1562*	0.0629	24.8733	0.0574	0.0679
LASSO-HARDRD-All	11.8869	0.2455*	0.1513*	23.5120	0.0907	0.0886
HARDRD-factor	11.9665	0.2455*	0.1187*	24.1687	0.0574	0.0886
LASSO-HARDRD-factor	11.9049	0.2455*	0.1466*	22.3516	1.0000**	1.0000**
			h=22			
HARDRD	11.4116	0.0651	0.0321	21.0709	0.0008	0.0578
HARDRD-Finance	11.4362	0.0651	0.0280	20.5207	0.0008	0.0824
LASSO-HARDRD-Finance	11.3540	0.0651	0.0321	20.8408	0.0008	0.0882
HARDRD-Macro	11.5875	0.0159	0.0145	20.5344	0.0008	0.1240*
LASSO-HARDRD-Macro	11.5167	0.0159	0.0193	20.1997	0.0438	0.2499*
HARDRD-Sentiment	11.5545	0.0159	0.0145	20.4734	0.0008	0.1240*
LASSO-HARDRD-Sentiment	11.4870	0.0159	0.0204	20.0689	0.0438	0.2499*
HARDRD-climate	11.4106	0.0651	0.0321	20.0467	0.2148*	0.2499*
LASSO-HARDRD-climate	11.3076	1.0000**	1.0000**	20.4998	0.0438	0.2276*
HARDRD-volatility	11.5003	0.0651	0.0283	20.4075	0.0438	0.2499*
LASSO-HARDRD-volatility	11.4169	0.0651	0.0321	19.4555	1.0000**	1.0000**
HARDRD-All	11.5798	0.0651	0.0247	20.1004	0.2148*	0.2499*
LASSO-HARDRD-All	11.5660	0.0159	0.0204	20.1482	0.0438	0.2499*
HARDRD-factor	11.4796	0.0159	0.0166	19.7891	0.2148*	0.2499*
LASSO-HARDRD-factor	11.4036	0.0651	0.0321	19.9484	0.2148*	0.2499*

Note: This table presents the pairwise comparisons of the out-of-sample forecasts from the benchmark HARDRD model against the extended model variations based on RMSE and Stein loss functions. The panels report the findings for the one-step, five-step and 22-step forecasts of realized covariance. The lower values of RMSE and Stein loss suggest the higher precision for the realized covariance forecasts. We also report the MCS results to test the significance of differences in forecast precision among different models. The columns denoted by "MCS_TR" and "MCS_TSQ" are the MCS test results given the range statistics,T_R and the semi-quadratic statistics T_{SQ} respectively. We denote the models that are included in the MCS at 10% and 25% confidence levels with * and ** respectively.

Table OA5: Robustness check of forecast precision with alternative forecast window (504 days).

	RMSE	MCS_TR	MCS_TSQ	Stein loss	MCS_TR	MCS_TSQ
			h=1			
HARDRD	12.4176	0.0027	0.0061	29.8671	0.0216	0.0460
HARDRD-Finance	12.4361	0.0027	0.0061	31.3235	0.0014	0.0139
LASSO-HARDRD-Finance	12.2469	0.0297	0.0297	27.8893	0.1797*	0.1904*
HARDRD-Macro	12.5902	0.0027	0.0061	31.7000	0.0216	0.0432
LASSO-HARDRD-Macro	12.4408	0.0027	0.0061	29.1745	0.0216	0.0695
HARDRD-Sentiment	12.5923	0.0027	0.0061	31.1655	0.0014	0.0170
LASSO-HARDRD-Sentiment	12.3205	0.0027	0.0061	28.5599	0.0216	0.0695
HARDRD-climate	12.4248	0.0027	0.0061	30.0054	0.0216	0.0607
LASSO-HARDRD-climate	12.2417	1.0000**	1.0000**	27.7978	0.3110**	0.3110**
HARDRD-volatility	12.4574	0.0027	0.0055	31.8156	0.0014	0.0105
LASSO-HARDRD-volatility	12.2756	0.0027	0.0061	27.6462	1.0000**	1.0000**
HARDRD-All	12.5818	0.0027	0.0055	35.8818	0.0216	0.0326
LASSO-HARDRD-All	12.4074	0.0027	0.0061	28.9019	0.1797*	0.1904*
HARDRD-factor	12.4958	0.0027	0.0061	31.0972	0.0019	0.0193
LASSO-HARDRD-factor	12.3461	0.0027	0.0061	28.9460	0.0216	0.0695
			h=5			
HARDRD	11.8630	0.1576*	0.1224*	28.3717	0.0188	0.0472
HARDRD-Finance	11.8209	0.2795**	0.1957*	29.9967	0.0679	0.0797
LASSO-HARDRD-Finance	11.7978	0.2795**	0.1957*	27.7276	0.0679	0.0818
HARDRD-Macro	11.8618	0.2383*	0.1510*	27.5742	0.0679	0.0869
LASSO-HARDRD-Macro	11.8291	0.2795**	0.1880*	27.6370	0.0679	0.0818
HARDRD-Sentiment	11.7998	0.2795**	0.2426*	27.6223	0.1129*	0.0869
LASSO-HARDRD-Sentiment	11.7398	1.0000**	1.0000**	27.8138	0.0679	0.0818
HARDRD-climate	11.8494	0.2795**	0.1880*	27.3121	0.1129*	0.0869
LASSO-HARDRD-climate	11.7696	0.4810**	0.4810**	27.1344	0.1129*	0.0869
HARDRD-volatility	11.8362	0.2795**	0.1957*	28.4340	0.0679	0.0818
LASSO-HARDRD-volatility	11.7895	0.2795**	0.2426*	26.3152	0.5203**	0.5203**
HARDRD-All	12.0170	0.1576*	0.0799	28.9182	0.0679	0.0663
LASSO-HARDRD-All	11.8323	0.2795**	0.1957*	27.3335	0.1129*	0.0869
HARDRD-factor	11.9005	0.2795**	0.1571*	28.1656	0.0679	0.0869
LASSO-HARDRD-factor	11.8472	0.2795**	0.1957*	25.8779	1.0000**	1.0000**
			h=22			
HARDRD	11.3166	0.0417	0.0304	25.3035	0.0009	0.0574
HARDRD-Finance	11.3423	0.0417	0.0286	24.5230	0.0009	0.1072*
LASSO-HARDRD-Finance	11.2696	0.0417	0.0315	24.9727	0.0009	0.0813
HARDRD-Macro	11.4811	0.0097	0.0195	24.6174	0.0009	0.1140*
LASSO-HARDRD-Macro	11.4146	0.0097	0.0195	24.2381	0.0429	0.2219*
HARDRD-Sentiment	11.4444	0.0097	0.0142	24.4926	0.0009	0.1140*
LASSO-HARDRD-Sentiment	11.3784	0.0097	0.0198	24.0367	0.0452	0.2219*
HARDRD-climate	11.3117	0.0417	0.0315	24.0003	0.2981**	0.3281**
LASSO-HARDRD-climate	11.2302	1.0000**	1.0000**	24.5835	0.0429	0.1848*
HARDRD-volatility	11.3932	0.0417	0.0304	24.3519	0.2981**	0.3281**
LASSO-HARDRD-volatility	11.3263	0.0417	0.0315	23.2374	1.0000**	1.0000**
HARDRD-All	11.4850	0.0097	0.0198	23.9523	0.4553**	0.3808**
LASSO-HARDRD-All	11.4775	0.0097	0.0198	24.0269	0.0429	0.2219*
HARDRD-factor	11.3858	0.0097	0.0172	23.5116	0.4553**	0.3808**
LASSO-HARDRD-factor	11.3112	0.0417	0.0315	23.8763	0.3394**	0.3281**

Note: This table presents the pairwise comparisons of the out-of-sample forecasts from the benchmark HARDRD model against the extended model variations based on RMSE and Stein loss functions. The panels report the findings for the one-step, five-step and 22-step forecasts of realized covariance. The lower values of RMSE and Stein loss suggest the higher precision for the realized covariance forecasts. We also report the MCS results to test the significance of differences in forecast precision among different models. The columns denoted by "MCS_TR" and "MCS_TSQ" are the MCS test results given the range statistics,T_R and the semi-quadratic statistics T_{SQ} respectively. We denote the models that are included in the MCS at 10% and 25% confidence levels with * and ** respectively.

Table OA6: Robustness check of portfolio analysis with alternative forecast window (126 days) of realized correlation matrix (h=1)

				c =0	c =1%	c =2%	c =0	c =1%	c =2%		γ=1			γ=10	
	GMVP	Turnover ratio	CO	Sharpe1	Sharpe2	Sharpe3	return1	return2	return3	EV1_1	EV1_2	EV1_3	EV2_1	EV2_2	EV2_3
								h=1							
HARDRD	0.1594	0.5806	0.4946	0.0495	0.0404	0.0313	7.9810	6.5179	5.0547	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARDRD-Finance	0.1609	0.5832	0.4940	0.0500	0.0409	0.0318	8.0808	6.6113	5.1417	0.1425	0.1332	0.1239	0.1221	0.1132	0.1043
LASSO-HARDRD-Finance	0.1516	0.5098	0.4718	0.0493	0.0414	0.0335	7.9919	6.7073	5.4226	0.0098	0.2684	0.5269	-0.0469	0.2119	0.4708
HARDRD-Macro	0.1697	0.3779	0.4722	0.0472	0.0413	0.0354	7.6147	6.6624	5.7101	-0.5284	0.2114	0.9512	-0.5074	0.2310	0.9696
LASSO-HARDRD-Macro	0.1615	0.4647	0.4638	0.0474	0.0402	0.0329	7.6608	6.4898	5.3188	-0.4639	-0.0409	0.3821	-0.4656	-0.0434	0.3790
HARDRD-Sentiment	0.1665	0.4151	0.4873	0.0474	0.0409	0.0345	7.6531	6.6070	5.5610	-0.4725	0.1315	0.7356	-0.4498	0.1540	0.7580
LASSO-HARDRD-Sentiment	0.1555	0.4861	0.4698	0.0496	0.0420	0.0345	8.0310	6.8060	5.5810	0.0666	0.4115	0.7564	0.0114	0.3565	0.7018
HARDRD-climate	0.1601	0.5880	0.4951	0.0474	0.0382	0.0290	7.6315	6.1498	4.6681	-0.5020	-0.5289	-0.5558	-0.4619	-0.4888	-0.5156
LASSO-HARDRD-climate	0.1513	0.5088	0.4714	0.0493	0.0413	0.0334	7.9723	6.6901	5.4079	-0.0164	0.2457	0.5078	-0.0528	0.2099	0.4726
HARDRD-volatility	0.1632	0.3146	0.4739	0.0475	0.0426	0.0377	7.6639	6.8712	6.0784	-0.4563	0.5145	1.4853	-0.4283	0.5404	1.5095
LASSO-HARDRD-volatility	0.1565	0.3752	0.4526	0.0474	0.0416	0.0358	7.6996	6.7541	5.8086	-0.4169	0.3327	1.0824	-0.5049	0.2438	0.9928
HARDRD-All	0.1650	0.3490	0.4877	0.0413	0.0359	0.0305	6.7684	5.8888	5.0092	-1.7763	-0.9310	-0.0857	-1.9649	-1.1187	-0.2722
LASSO-HARDRD-All	0.1592	0.3945	0.4547	0.0488	0.0427	0.0365	7.9065	6.9124	5.9182	-0.1137	0.5655	1.2448	-0.1687	0.5098	1.1887
HARDRD-factor	0.1648	0.3165	0.4721	0.0494	0.0444	0.0395	7.9873	7.1897	6.3921	0.0074	0.9711	1.9348	-0.0091	0.9521	1.9138
LASSO-HARDRD-factor	0.1581	0.4884	0.4633	0.0481	0.0405	0.0329	7.7835	6.5526	5.3218	-0.2911	0.0453	0.3817	-0.3383	-0.0026	0.3332
								h=5							
HARDRD	0.1874	0.1506	0.4923	0.0968	0.0915	0.0861	6.8448	6.4652	6.0857	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARDRD-Finance	0.1845	0.1487	0.4939	0.1036	0.0983	0.0930	7.3291	6.9543	6.5796	0.7012	0.7081	0.7151	0.6983	0.7055	0.7127
LASSO-HARDRD-Finance	0.1842	0.1410	0.4834	0.0996	0.0945	0.0895	7.0167	6.6614	6.3060	0.2498	0.2850	0.3202	0.2576	0.2938	0.3301
HARDRD-Macro	0.1903	0.1124	0.4840	0.0963	0.0922	0.0882	6.7458	6.4624	6.1790	-0.1405	-0.0012	0.1380	-0.1135	0.0258	0.1650
LASSO-HARDRD-Macro	0.1843	0.1363	0.4780	0.0987	0.0938	0.0889	6.9674	6.6240	6.2805	0.1788	0.2312	0.2835	0.1899	0.2427	0.2956
HARDRD-Sentiment	0.1899	0.1333	0.4928	0.1024	0.0976	0.0929	7.2677	6.9319	6.5960	0.6115	0.6748	0.7380	0.6012	0.6645	0.7278
LASSO-HARDRD-Sentiment	0.1851	0.1505	0.4870	0.1036	0.0983	0.0929	7.3638	6.9844	6.6051	0.7497	0.7501	0.7505	0.7298	0.7317	0.7337
HARDRD-climate	0.1901	0.1617	0.4929	0.1018	0.0961	0.0903	7.2162	6.8087	6.4011	0.5367	0.4961	0.4556	0.5241	0.4837	0.4433
LASSO-HARDRD-climate	0.1833	0.1106	0.4793	0.0976	0.0936	0.0897	6.8778	6.5990	6.3202	0.0492	0.1951	0.3411	0.0616	0.2081	0.3545
HARDRD-volatility	0.1873	0.1391	0.4871	0.0967	0.0917	0.0867	6.8019	6.4514	6.1008	-0.0600	-0.0181	0.0239	-0.0408	0.0012	0.0432
LASSO-HARDRD-volatility	0.1844	0.1155	0.4748	0.0995	0.0954	0.0912	7.0162	6.7251	6.4340	0.2493	0.3774	0.5055	0.2585	0.3871	0.5158
HARDRD-All	0.1945	0.2037	0.5001	0.1161	0.1089	0.1017	8.2694	7.7561	7.2428	2.0592	1.8655	1.6718	2.0189	1.8260	1.6332
LASSO-HARDRD-All	0.1873	0.1769	0.4860	0.1202	0.1139	0.1077	8.5275	8.0817	7.6359	2.4351	2.3391	2.2431	2.4135	2.3182	2.2228
HARDRD-factor	0.1919	0.2022	0.4900	0.1035	0.0962	0.0890	7.2836	6.7741	6.2645	0.6382	0.4499	0.2616	0.6630	0.4746	0.2861
LASSO-HARDRD-factor	0.1891	0.1279	0.4770	0.1029	0.0983	0.0937	7.2361	6.9138	6.5916	0.5691	0.6520	0.7350	0.5903	0.6736	0.7569
								h=22							

HARDRD	0.2131	0.1010	0.5069	0.2341	0.2256	0.2171	6.9161	6.6617	6.4072	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARDRD-Finance	0.2084	0.1080	0.5255	0.2167	0.2077	0.1986	6.4586	6.1865	5.9143	-0.6633	-0.6889	-0.7145	-0.6693	-0.6949	-0.7206
LASSO-HARDRD-Finance	0.2097	0.1136	0.4967	0.2224	0.2128	0.2032	6.6050	6.3186	6.0323	-0.4510	-0.4973	-0.5435	-0.4545	-0.5012	-0.5479
HARDRD-Macro	0.2223	0.0887	0.5146	0.2202	0.2128	0.2053	6.5359	6.3123	6.0887	-0.5505	-0.5058	-0.4612	-0.5480	-0.5038	-0.4596
LASSO-HARDRD-Macro	0.2201	0.1103	0.4991	0.2161	0.2068	0.1974	6.4226	6.1447	5.8668	-0.7148	-0.7488	-0.7828	-0.7142	-0.7486	-0.7831
HARDRD-Sentiment	0.2168	0.1044	0.5077	0.2228	0.2142	0.2055	6.6714	6.4082	6.1449	-0.3558	-0.3685	-0.3812	-0.3679	-0.3809	-0.3938
LASSO-HARDRD-Sentiment	0.2154	0.1195	0.4980	0.2389	0.2288	0.2186	7.0270	6.7257	6.4245	0.1609	0.0931	0.0253	0.1642	0.0963	0.0284
HARDRD-climate	0.2166	0.1068	0.5049	0.2418	0.2328	0.2238	7.1408	6.8717	6.6026	0.3254	0.3042	0.2830	0.3257	0.3043	0.2829
LASSO-HARDRD-climate	0.2075	0.1042	0.4950	0.2257	0.2170	0.2083	6.7280	6.4654	6.2028	-0.2730	-0.2848	-0.2967	-0.2775	-0.2898	-0.3022
HARDRD-volatility	0.2199	0.1019	0.5117	0.2448	0.2362	0.2276	7.2107	6.9538	6.6969	0.4270	0.4235	0.4199	0.4303	0.4262	0.4221
LASSO-HARDRD-volatility	0.2139	0.1172	0.4935	0.2461	0.2360	0.2260	7.1801	6.8848	6.5895	0.3824	0.3232	0.2640	0.3834	0.3236	0.2638
HARDRD-All	0.2141	0.1323	0.5253	0.2261	0.2153	0.2045	6.8805	6.5472	6.2139	-0.0511	-0.1654	-0.2797	-0.0461	-0.1616	-0.2770
LASSO-HARDRD-All	0.2140	0.1220	0.5152	0.2208	0.2107	0.2007	6.7025	6.3951	6.0878	-0.3096	-0.3863	-0.4630	-0.3112	-0.3888	-0.4664
HARDRD-factor	0.2085	0.0942	0.5333	0.2248	0.2169	0.2090	6.7224	6.4851	6.2477	-0.2809	-0.2561	-0.2314	-0.2835	-0.2592	-0.2350
LASSO-HARDRD-factor	0.2116	0.1135	0.4932	0.2163	0.2068	0.1972	6.4406	6.1547	5.8688	-0.6886	-0.7341	-0.7796	-0.6872	-0.7326	-0.7781

Note: The table reports the economic evaluation results for h-step ahead forecasts obtained from competing forecasting models. We report the GMVP, the average turnover ratio, the average concentration ration (CO), and annualized average out-of-sample return given three transaction cost rate c= 0, 1%, 2%. We also report the Sharpe Ratios (SR) which are computed based on three different transaction costs, i.e. (c= 0, 1%, 2%) as well as the Economic Value (EV) of the portfolios are over the benchmark HARDRD model is obtained using two risk aversion rates, γ=1 and γ=10.

Table OA7: Robustness check of portfolio analysis with alternative forecast window (252 days) of realized correlation matrix (h=1)

				c =0	c =1%	c =2%	c =0	c =1%	c =2%		γ=1			γ=10	
	GMVP	Turnover ratio	CO	Sharpe1	Sharpe2	Sharpe3	return1	return2	return3	EV1_1	EV1_2	EV1_3	EV2_1	EV2_2	EV2_3
								h=1							
HARDRD	0.1608	0.5608	0.4875	0.0483	0.0395	0.0307	7.7352	6.3219	4.9087	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARDRD-Finance	0.1624	0.5630	0.4875	0.0488	0.0400	0.0311	7.8303	6.4114	4.9926	0.1352	0.1272	0.1191	0.1118	0.1040	0.0963
LASSO-HARDRD-Finance	0.1526	0.4924	0.4649	0.0483	0.0406	0.0328	7.7562	6.5154	5.2747	0.0253	0.2753	0.5253	-0.0226	0.2282	0.4791
HARDRD-Macro	0.1719	0.3648	0.4661	0.0467	0.0409	0.0352	7.4783	6.5589	5.6396	-0.3707	0.3448	1.0604	-0.3572	0.3596	1.0766
LASSO-HARDRD-Macro	0.1636	0.4510	0.4555	0.0468	0.0397	0.0326	7.4825	6.3461	5.2097	-0.3645	0.0366	0.4376	-0.3505	0.0510	0.4527
HARDRD-Sentiment	0.1691	0.4025	0.4796	0.0465	0.0402	0.0338	7.4395	6.4252	5.4108	-0.4246	0.1533	0.7313	-0.3902	0.1895	0.7694
LASSO-HARDRD-Sentiment	0.1570	0.4688	0.4625	0.0490	0.0416	0.0343	7.8623	6.6808	5.4993	0.1799	0.5157	0.8516	0.1410	0.4779	0.8149
HARDRD-climate	0.1616	0.5680	0.4878	0.0469	0.0380	0.0290	7.4976	6.0663	4.6351	-0.3410	-0.3670	-0.3930	-0.3108	-0.3364	-0.3621
LASSO-HARDRD-climate	0.1523	0.4910	0.4645	0.0482	0.0405	0.0328	7.7314	6.4942	5.2569	-0.0085	0.2466	0.5017	-0.0361	0.2200	0.4762
HARDRD-volatility	0.1648	0.3018	0.4690	0.0474	0.0426	0.0379	7.5842	6.8237	6.0632	-0.2168	0.7288	1.6744	-0.1983	0.7480	1.6947
LASSO-HARDRD-volatility	0.1579	0.3646	0.4460	0.0469	0.0412	0.0355	7.5561	6.6373	5.7184	-0.2680	0.4484	1.1647	-0.3477	0.3697	1.0873
HARDRD-All	0.1669	0.3383	0.4804	0.0418	0.0366	0.0314	6.8026	5.9502	5.0977	-1.3697	-0.5571	0.2556	-1.5486	-0.7323	0.0844
LASSO-HARDRD-All	0.1613	0.3844	0.4471	0.0485	0.0425	0.0365	7.7954	6.8267	5.8579	0.0828	0.7267	1.3707	0.0409	0.6862	1.3318
HARDRD-factor	0.1665	0.3065	0.4660	0.0492	0.0444	0.0395	7.8917	7.1192	6.3468	0.2244	1.1527	2.0810	0.2039	1.1326	2.0618
LASSO-HARDRD-factor	0.1597	0.4741	0.4565	0.0475	0.0401	0.0327	7.6322	6.4375	5.2428	-0.1531	0.1635	0.4802	-0.1899	0.1269	0.4438
								h=5							
HARDRD	0.1863	0.1450	0.4896	0.1002	0.0951	0.0899	7.1128	6.7475	6.3823	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARDRD-Finance	0.1832	0.1429	0.4915	0.1063	0.1012	0.0962	7.5474	7.1873	6.8273	0.6292	0.6368	0.6444	0.6276	0.6355	0.6434
LASSO-HARDRD-Finance	0.1832	0.1354	0.4808	0.1032	0.0984	0.0936	7.2973	6.9561	6.6148	0.2684	0.3034	0.3383	0.2798	0.3159	0.3520
HARDRD-Macro	0.1893	0.1087	0.4814	0.1002	0.0963	0.0924	7.0332	6.7591	6.4851	-0.1122	0.0201	0.1523	-0.0814	0.0512	0.1837
LASSO-HARDRD-Macro	0.1835	0.1329	0.4748	0.1037	0.0990	0.0942	7.3409	7.0060	6.6712	0.3317	0.3759	0.4200	0.3451	0.3900	0.4349
HARDRD-Sentiment	0.1879	0.1286	0.4912	0.1044	0.0999	0.0953	7.4460	7.1220	6.7980	0.4815	0.5413	0.6012	0.4719	0.5319	0.5919
LASSO-HARDRD-Sentiment	0.1837	0.1452	0.4844	0.1058	0.1006	0.0955	7.5418	7.1757	6.8097	0.6197	0.6188	0.6179	0.6050	0.6057	0.6063
HARDRD-climate	0.1889	0.1555	0.4894	0.1036	0.0981	0.0926	7.3716	6.9798	6.5880	0.3737	0.3353	0.2970	0.3640	0.3256	0.2872
LASSO-HARDRD-climate	0.1823	0.1067	0.4767	0.1014	0.0976	0.0938	7.1722	6.9034	6.6346	0.0875	0.2274	0.3673	0.1020	0.2427	0.3835
HARDRD-volatility	0.1858	0.1334	0.4866	0.1005	0.0957	0.0910	7.1053	6.7692	6.4331	-0.0093	0.0330	0.0753	0.0055	0.0478	0.0902
LASSO-HARDRD-volatility	0.1834	0.1113	0.4732	0.1026	0.0987	0.0947	7.2699	6.9895	6.7090	0.2284	0.3514	0.4744	0.2366	0.3605	0.4843
HARDRD-All	0.1930	0.1963	0.4992	0.1153	0.1084	0.1014	8.2201	7.7255	7.2309	1.6012	1.4139	1.2266	1.5764	1.3895	1.2025
LASSO-HARDRD-All	0.1856	0.1710	0.4846	0.1195	0.1134	0.1074	8.4835	8.0525	7.6215	1.9847	1.8896	1.7945	1.9797	1.8848	1.7899
HARDRD-factor	0.1903	0.1962	0.4891	0.1063	0.0993	0.0923	7.5081	7.0137	6.5193	0.5755	0.3883	0.2012	0.6030	0.4151	0.2273
LASSO-HARDRD-factor	0.1880	0.1240	0.4754	0.1061	0.1017	0.0973	7.4790	7.1666	6.8542	0.5334	0.6100	0.6867	0.5620	0.6391	0.7162
								h=22							
HARDRD	0.2103	0.0964	0.5010	0.2393	0.2312	0.2230	7.1034	6.8605	6.6175	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

HARDRD-Finance	0.2054	0.1033	0.5198	0.2232	0.2146	0.2059	6.6804	6.4202	6.1599	-0.6133	-0.6383	-0.6634	-0.6189	-0.6441	-0.6693
LASSO-HARDRD-Finance	0.2072	0.1108	0.4913	0.2301	0.2207	0.2113	6.8497	6.5704	6.2911	-0.3678	-0.4205	-0.4732	-0.3707	-0.4239	-0.4771
HARDRD-Macro	0.2201	0.0855	0.5094	0.2295	0.2223	0.2151	6.8403	6.6248	6.4093	-0.3809	-0.3412	-0.3015	-0.3793	-0.3399	-0.3005
LASSO-HARDRD-Macro	0.2182	0.1074	0.4922	0.2249	0.2158	0.2067	6.7060	6.4354	6.1648	-0.5755	-0.6156	-0.6556	-0.5746	-0.6150	-0.6555
HARDRD-Sentiment	0.2153	0.1005	0.4997	0.2288	0.2204	0.2119	6.8624	6.6091	6.3558	-0.3501	-0.3652	-0.3802	-0.3595	-0.3749	-0.3902
LASSO-HARDRD-Sentiment	0.2136	0.1159	0.4902	0.2463	0.2364	0.2265	7.2564	6.9644	6.6725	0.2222	0.1512	0.0802	0.2277	0.1565	0.0852
HARDRD-climate	0.2137	0.1023	0.4983	0.2467	0.2380	0.2293	7.3284	7.0705	6.8126	0.3259	0.3042	0.2825	0.3264	0.3045	0.2826
LASSO-HARDRD-climate	0.2052	0.1009	0.4894	0.2338	0.2253	0.2168	6.9774	6.7232	6.4690	-0.1828	-0.1992	-0.2155	-0.1851	-0.2019	-0.2188
HARDRD-volatility	0.2168	0.0973	0.5101	0.2493	0.2410	0.2328	7.3882	7.1429	6.8976	0.4128	0.4093	0.4059	0.4158	0.4118	0.4077
LASSO-HARDRD-volatility	0.2111	0.1138	0.4885	0.2505	0.2407	0.2309	7.3517	7.0650	6.7782	0.3597	0.2962	0.2328	0.3605	0.2963	0.2322
HARDRD-All	0.2119	0.1285	0.5229	0.2382	0.2276	0.2170	7.2614	6.9375	6.6137	0.2298	0.1125	-0.0049	0.2389	0.1201	0.0013
LASSO-HARDRD-All	0.2117	0.1186	0.5112	0.2314	0.2216	0.2119	7.0463	6.7475	6.4486	-0.0827	-0.1637	-0.2448	-0.0823	-0.1644	-0.2465
HARDRD-factor	0.2056	0.0904	0.5276	0.2293	0.2217	0.2142	6.8929	6.6650	6.4372	-0.3054	-0.2835	-0.2617	-0.3094	-0.2879	-0.2664
LASSO-HARDRD-factor	0.2094	0.1091	0.4884	0.2254	0.2162	0.2070	6.7202	6.4453	6.1703	-0.5547	-0.6011	-0.6474	-0.5521	-0.5984	-0.6447

Note: The table reports the economic evaluation results for h-step ahead forecasts obtained from competing forecasting models. We report the GMVP, the average turnover ratio, the average concentration ration (CO), and annualized average out-of-sample return given three transaction cost rate c= 0, 1%, 2%. We also report the Sharpe Ratios (SR) which are computed based on three different transaction costs, i.e. (c= 0, 1%, 2%) as well as the Economic Value (EV) of the portfolios are over the benchmark HARDRD model is obtained using two risk aversion rates, γ=1 and γ=10.

Table OA8: Robustness check of portfolio analysis with alternative forecast window (504 days) of realized correlation matrix (h=1)

	GMVP	turnover ratio	CO	c =0 Sharpe1	c =1% Sharpe2	c =2% Sharpe3	c =0 return1	c =1% return2	c =2% return3	γ=1 EV1_1	γ=1 EV1_2	γ=1 EV1_3	γ=10 EV2_1	γ=10 EV2_2	γ=10 EV2_3
							h=1								
HARDRD	0.1580	0.5596	0.5040	0.0465	0.0378	0.0290	7.4944	6.0841	4.6739	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARDRD-Finance	0.1593	0.5619	0.5044	0.0469	0.0381	0.0293	7.5547	6.1387	4.7228	0.0862	0.0778	0.0695	0.0739	0.0657	0.0576
LASSO-HARDRD-Finance	0.1513	0.4928	0.4787	0.0462	0.0385	0.0308	7.4540	6.2122	4.9705	-0.0613	0.1828	0.4269	-0.0885	0.1569	0.4025
HARDRD-Macro	0.1697	0.3631	0.4816	0.0456	0.0399	0.0342	7.3449	6.4298	5.5147	-0.2154	0.5021	1.2196	-0.2046	0.5160	1.2370
LASSO-HARDRD-Macro	0.1620	0.4545	0.4675	0.0439	0.0368	0.0296	7.0562	5.9109	4.7656	-0.6312	-0.2474	0.1364	-0.5979	-0.2133	0.1714
HARDRD-Sentiment	0.1662	0.3983	0.4965	0.0463	0.0401	0.0338	7.4421	6.4383	5.4345	-0.0696	0.5195	1.1085	-0.0118	0.5807	1.1734
LASSO-HARDRD-Sentiment	0.1553	0.4679	0.4766	0.0462	0.0388	0.0315	7.4329	6.2537	5.0745	-0.0899	0.2449	0.5797	-0.0981	0.2383	0.5748
HARDRD-climate	0.1586	0.5657	0.5051	0.0455	0.0366	0.0278	7.3210	5.8954	4.4699	-0.2490	-0.2712	-0.2934	-0.2288	-0.2506	-0.2724
LASSO-HARDRD-climate	0.1511	0.4903	0.4777	0.0460	0.0384	0.0307	7.4201	6.1845	4.9488	-0.1087	0.1443	0.3974	-0.1193	0.1352	0.3897
HARDRD-volatility	0.1615	0.2976	0.4876	0.0460	0.0414	0.0367	7.4142	6.6644	5.9145	-0.1141	0.8428	1.7997	-0.0948	0.8655	1.8262
LASSO-HARDRD-volatility	0.1566	0.3638	0.4587	0.0440	0.0383	0.0326	7.1034	6.1867	5.2701	-0.5708	0.1444	0.8595	-0.6131	0.1045	0.8225
HARDRD-All	0.1636	0.3323	0.4964	0.0428	0.0377	0.0326	7.0106	6.1733	5.3359	-0.7216	0.1088	0.9392	-0.9192	-0.0833	0.7529
LASSO-HARDRD-All	0.1601	0.3846	0.4587	0.0454	0.0394	0.0334	7.3103	6.3410	5.3717	-0.2655	0.3734	1.0124	-0.2548	0.3867	1.0285
HARDRD-factor	0.1636	0.3048	0.4830	0.0476	0.0428	0.0381	7.6853	6.9171	6.1489	0.2746	1.2048	2.1350	0.2564	1.1893	2.1225
LASSO-HARDRD-factor	0.1577	0.4737	0.4712	0.0448	0.0374	0.0300	7.2274	6.0336	4.8398	-0.3881	-0.0746	0.2390	-0.4020	-0.0876	0.2269
							h=5								
HARDRD	0.1808	0.1458	0.5121	0.0938	0.0887	0.0836	6.7390	6.3717	6.0044	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARDRD-Finance	0.1776	0.1418	0.5143	0.0993	0.0943	0.0893	7.1397	6.7823	6.4249	0.5802	0.5946	0.6090	0.5791	0.5938	0.6086
LASSO-HARDRD-Finance	0.1781	0.1339	0.5041	0.0964	0.0917	0.0870	6.8961	6.5586	6.2210	0.2290	0.2723	0.3155	0.2432	0.2875	0.3318
HARDRD-Macro	0.1833	0.1083	0.5070	0.0937	0.0898	0.0860	6.6632	6.3902	6.1173	-0.1069	0.0299	0.1666	-0.0790	0.0584	0.1959
LASSO-HARDRD-Macro	0.1781	0.1292	0.4983	0.0972	0.0927	0.0881	6.9661	6.6406	6.3151	0.3305	0.3912	0.4519	0.3462	0.4076	0.4689
HARDRD-Sentiment	0.1819	0.1274	0.5145	0.0992	0.0947	0.0903	7.1751	6.8539	6.5328	0.6302	0.6971	0.7640	0.6168	0.6841	0.7513
LASSO-HARDRD-Sentiment	0.1785	0.1401	0.5072	0.0994	0.0945	0.0896	7.1732	6.8203	6.4674	0.6277	0.6487	0.6697	0.6158	0.6382	0.6606
HARDRD-climate	0.1835	0.1548	0.5121	0.0968	0.0913	0.0859	6.9688	6.5786	6.1884	0.3319	0.2988	0.2656	0.3233	0.2900	0.2568
LASSO-HARDRD-climate	0.1775	0.1054	0.4989	0.0947	0.0910	0.0873	6.7784	6.5127	6.2470	0.0589	0.2062	0.3535	0.0773	0.2257	0.3740
HARDRD-volatility	0.1804	0.1340	0.5105	0.0949	0.0901	0.0854	6.7957	6.4582	6.1206	0.0834	0.1264	0.1695	0.0950	0.1382	0.1814
LASSO-HARDRD-volatility	0.1785	0.1098	0.4963	0.0962	0.0924	0.0885	6.8996	6.6228	6.3461	0.2337	0.3650	0.4963	0.2446	0.3770	0.5094
HARDRD-All	0.1872	0.1929	0.5218	0.1085	0.1018	0.0951	7.8248	7.3385	6.8523	1.5708	1.3986	1.2263	1.5541	1.3818	1.2094
LASSO-HARDRD-All	0.1804	0.1686	0.5072	0.1131	0.1071	0.1012	8.1180	7.6932	7.2684	1.9978	1.9146	1.8313	2.0032	1.9196	1.8360
HARDRD-factor	0.1844	0.1979	0.5125	0.1015	0.0945	0.0876	7.2614	6.7627	6.2641	0.7598	0.5694	0.3790	0.7894	0.5977	0.4059
LASSO-HARDRD-factor	0.1828	0.1226	0.4971	0.1011	0.0967	0.0924	7.1993	6.8903	6.5814	0.6705	0.7551	0.8397	0.7074	0.7925	0.8776
							h=22								
HARDRD	0.2025	0.0952	0.5262	0.2265	0.2186	0.2106	6.8013	6.5613	6.3213	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARDRD-Finance	0.1972	0.1022	0.5464	0.2088	0.2003	0.1919	6.3374	6.0798	5.8223	-0.6726	-0.6981	-0.7235	-0.6794	-0.7051	-0.7307

LASSO-HARDRD-Finance	0.1998	0.1114	0.5173	0.2159	0.2066	0.1973	6.5068	6.2260	5.9452	-0.4269	-0.4861	-0.5454	-0.4306	-0.4906	-0.5506
HARDRD-Macro	0.2120	0.0856	0.5365	0.2167	0.2096	0.2024	6.5430	6.3274	6.1117	-0.3739	-0.3387	-0.3035	-0.3730	-0.3380	-0.3029
LASSO-HARDRD-Macro	0.2106	0.1072	0.5174	0.2104	0.2015	0.1925	6.3529	6.0827	5.8124	-0.6494	-0.6933	-0.7372	-0.6495	-0.6940	-0.7385
HARDRD-Sentiment	0.2072	0.1006	0.5249	0.2116	0.2033	0.1950	6.4491	6.1956	5.9421	-0.5113	-0.5309	-0.5505	-0.5227	-0.5428	-0.5628
LASSO-HARDRD-Sentiment	0.2062	0.1154	0.5151	0.2325	0.2228	0.2131	6.9188	6.6281	6.3374	0.1709	0.0974	0.0238	0.1776	0.1035	0.0294
HARDRD-climate	0.2059	0.1018	0.5240	0.2351	0.2266	0.2181	7.0514	6.7949	6.5383	0.3626	0.3386	0.3146	0.3659	0.3416	0.3172
LASSO-HARDRD-climate	0.1982	0.1018	0.5146	0.2207	0.2122	0.2037	6.6489	6.3924	6.1358	-0.2208	-0.2448	-0.2689	-0.2215	-0.2462	-0.2709
HARDRD-volatility	0.2090	0.0983	0.5379	0.2372	0.2290	0.2208	7.1102	6.8625	6.6148	0.4480	0.4368	0.4255	0.4527	0.4408	0.4289
LASSO-HARDRD-volatility	0.2039	0.1147	0.5149	0.2355	0.2258	0.2161	7.0046	6.7157	6.4267	0.2946	0.2235	0.1524	0.2943	0.2222	0.1500
HARDRD-All	0.2039	0.1315	0.5515	0.2298	0.2192	0.2085	7.1002	6.7689	6.4377	0.4345	0.3020	0.1696	0.4483	0.3142	0.1800
LASSO-HARDRD-All	0.2040	0.1210	0.5390	0.2202	0.2104	0.2005	6.7918	6.4868	6.1819	-0.0134	-0.1077	-0.2019	-0.0102	-0.1058	-0.2014
HARDRD-factor	0.1975	0.0902	0.5549	0.2129	0.2054	0.1980	6.4869	6.2594	6.0320	-0.4559	-0.4378	-0.4197	-0.4610	-0.4432	-0.4254
LASSO-HARDRD-factor	0.2025	0.1079	0.5129	0.2121	0.2031	0.1941	6.3892	6.1173	5.8454	-0.5966	-0.6429	-0.6892	-0.5942	-0.6408	-0.6874

Note: The table reports the economic evaluation results for h-step ahead forecasts obtained from competing forecasting models. We report the GMVP, the average turnover ratio, the average concentration ration (CO), and annualized average out-of-sample return given three transaction cost rate c= 0, 1%, 2%. We also report the Sharpe Ratios (SR) which are computed based on three different transaction costs, i.e. (c= 0, 1%, 2%) as well as the Economic Value (EV) of the portfolios are over the benchmark HARDRD model is obtained using two risk aversion rates, γ=1 and γ=10.

Figure OA1: Cumulative portfolio returns for different MHAR models (long out-of-sample).